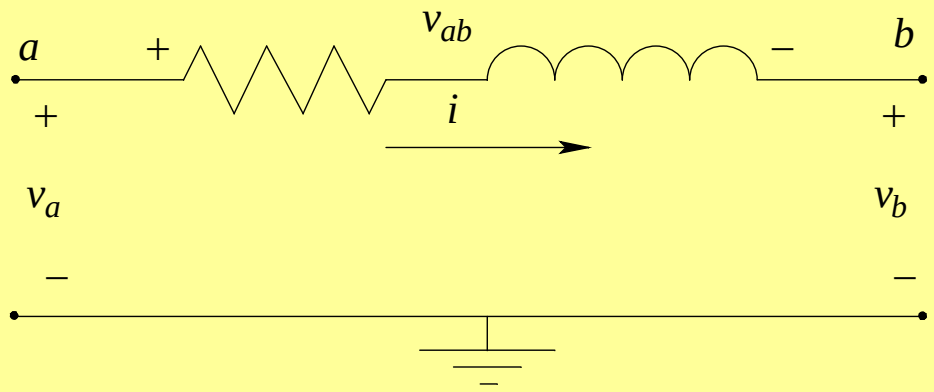
Review of Basic Electric Circuits

- Convention and Symbols
- Use of Phasors
- Power, Reactive Power, Power Factor
- Inductive and Capacitive Loads

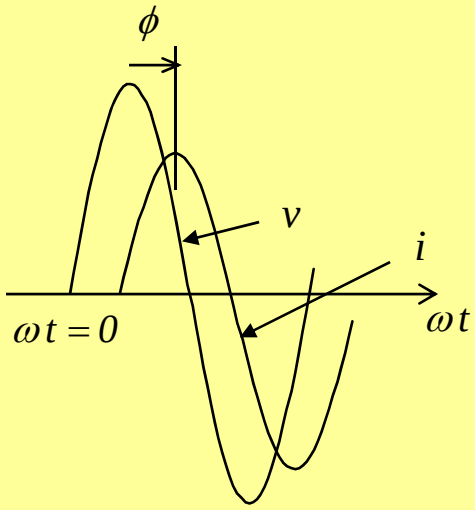
Conventions

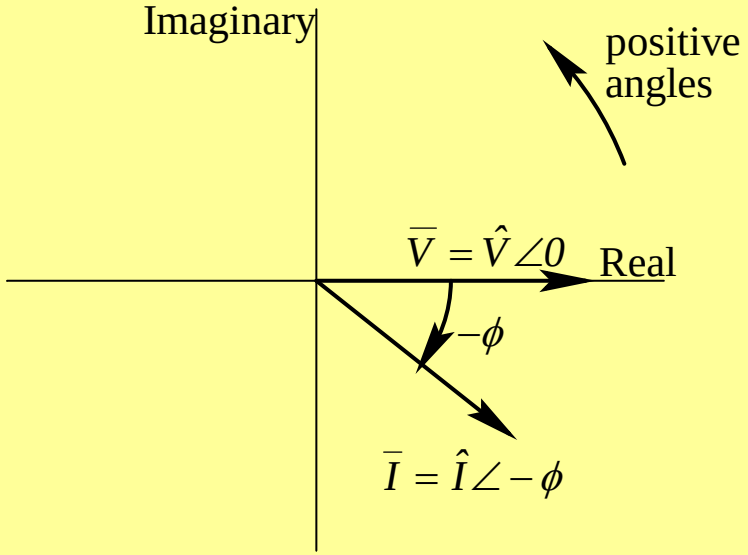
- MKS (SI) Units
- lower case *v* and *i* for instantaneous quantities
- upper case *V* and *I* for average and rms
- voltage and current subscripts



- voltage polarities and current directions

Phasor Domain Representation for Sinusoidal Steady State AC

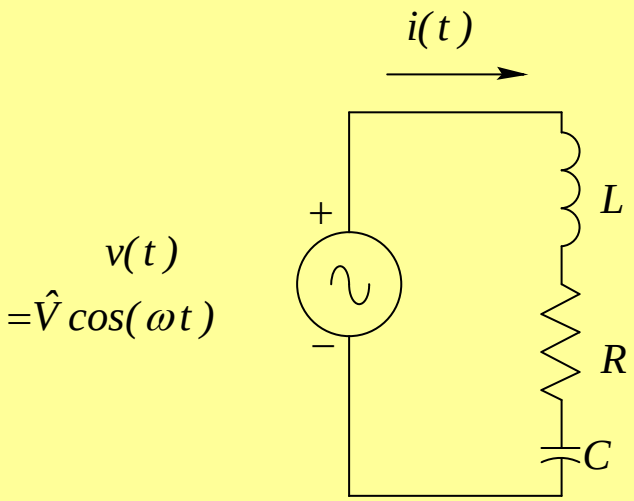




$$v(t) = \hat{V}\cos(\omega t) \quad \Leftrightarrow \quad \bar{V} = \hat{V}\angle 0$$

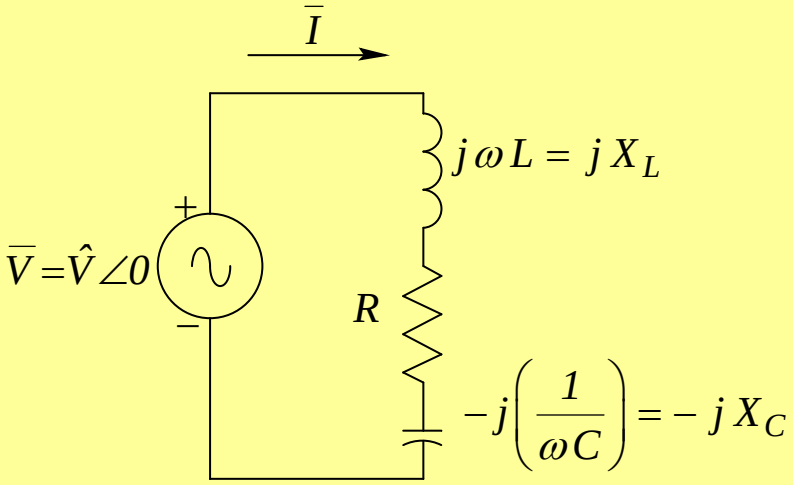
$$i(t) = \hat{I}\cos(\omega t - \phi) \quad \Leftrightarrow \quad \bar{I} = \hat{I}\angle -\phi$$

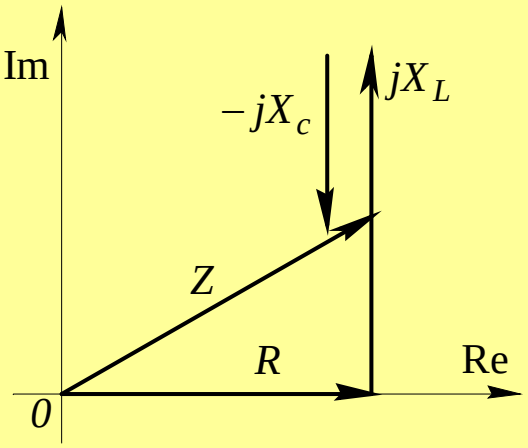
Time-Domain Analysis



$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\cdot dt \;=\; \hat{V}\cos(\omega t)$$

Phasor Domain Analysis



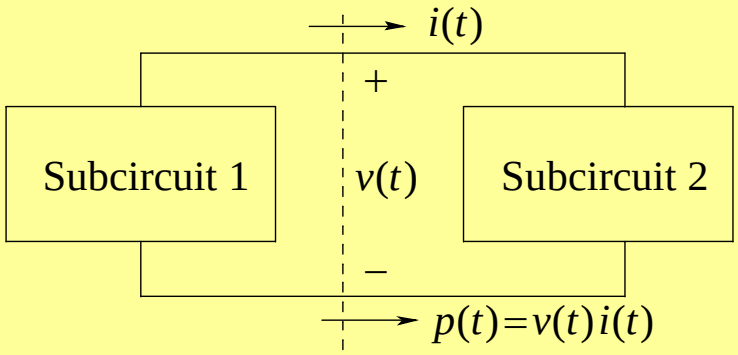


$$Z = R + jX_L - jX_C = |Z|\angle\phi$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad ; \quad \phi = \tan^{-1}\left[\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]$$

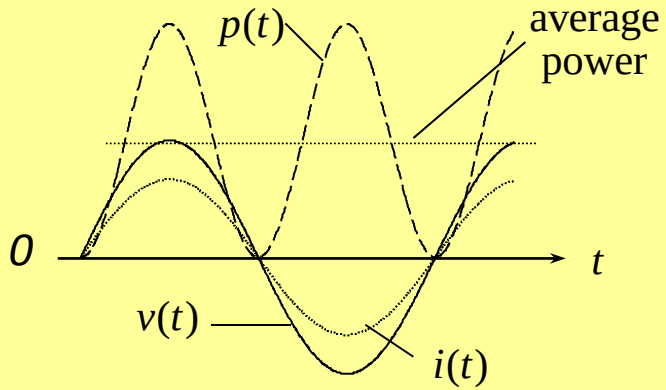
$$i(t) = \frac{\hat{V}}{|Z|}\cos(\omega t - \phi) \quad \Leftrightarrow \quad \bar{I} = \frac{\bar{V}}{Z} = \frac{\hat{V}}{|Z|}\angle -\phi$$

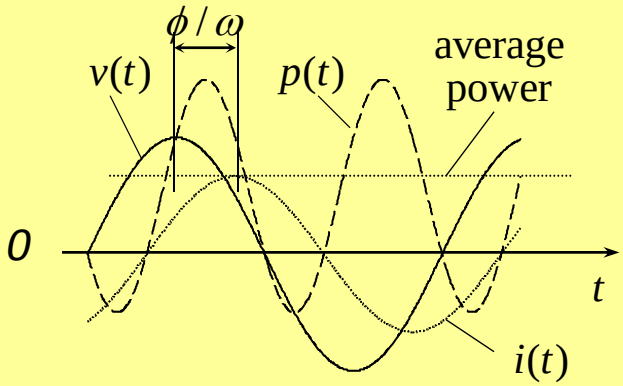
Instantaneous Power



$$v(t) = \hat{V}\cos(\omega t + \phi_v)$$

$$i(t) = \hat{I}\cos(\omega t + \phi_i)$$





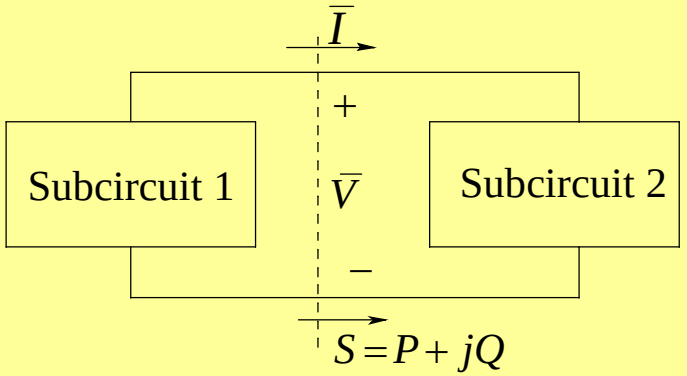
- *v* and *i* in phase ($\phi_v = \phi_i$)
- power flows in one direction
- maximum average power for given *V* and *I*

- *v* and *i* out of phase ($\phi_v \neq \phi_i$)
- power flow reverses periodically
- average power lower than maximum possible

Real Power, Reactive Power and Power factor

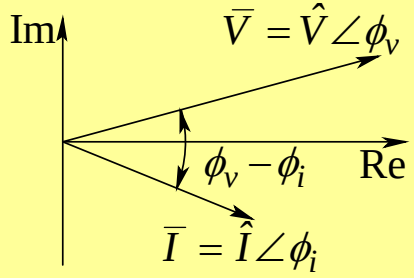
- Complex Power

$$S = \bar{V}\ \bar{I}^{*} \quad (S \text{ is a complex number})$$
$$= V\ I \angle(\phi_v - \phi_i) = V\ I \angle\phi$$
$$S = P + jQ = |S| \angle\phi$$



- Real Power (average power)

$$P = V\ I\ \cos\phi \qquad [W]$$

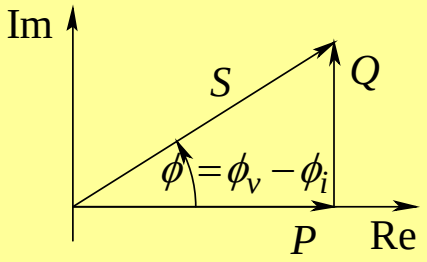


- Reactive Power

$$Q = V\ I\ \sin\phi \qquad [VAR]$$

- Apparent Power

$$|S| = \sqrt{P^2 + Q^2} = VI \qquad [VA]$$



- Power Factor

$$PF = \frac{P}{|S|} = \frac{P}{V\ I} = \cos\phi$$

Inductive Load

- The impedance is $Z = |Z| \angle \phi$ where ϕ is positive
- The current lags the voltage by the impedance angle ϕ
- Corresponds to a lagging power factor of operation
- In the power triangle, the same angle ϕ relates P,Q and $|S|$
- An inductive load draws positive reactive power (VARs)
- Most loads are inductive, particularly motors and transformers

Summary

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