Module 3 : Sequence Components and Fault Analysis

Lecture 11 : Sequence Components (Tutorial)

Objectives

In this lecture we will solve some tutorial problems to

- To extract sequence components from an unbalanced phasor.
- Define sequence transformation with 'b' as reference phasor.
- Analyze the effect of changing reference phasor.
- Find out fault currents for S-L-G, L-L and L-L-G faults.

1. The currents in a $3-\phi$ unbalanced system are given by

$$\overrightarrow{I_a} = (10+j4)A,\ \overrightarrow{I_b} = (11-j9)A,\ \overrightarrow{I_c} = (-15+j9)A$$

Calculate the zero, positive and negative sequence currents.

Ans:

$$\overrightarrow{I_{a0}} = \frac{1}{3}(\overrightarrow{I_a}+\overrightarrow{I_b}+\overrightarrow{I_c})$$

$$= \frac{1}{3}(10+j4+11-j9-15+9j)$$

$$= \frac{1}{3}(6+j4) = (2+j1.33)A$$

$$\overrightarrow{I_{a1}} = \frac{1}{3}(\overrightarrow{I_a}+a\overrightarrow{I_b}+a^2\overrightarrow{I_c})$$

where $a = -0.5+j0.866$

$$a^2 = -0.5-j0.866$$

$$= \frac{1}{3}((10+j4)+(-0.5+j0.866)(11-j9)+(-0.5-j0.866)(-15+j9))$$

$$= \frac{1}{3}(10+j4+2.294+j14.026+15.294+j8.49)$$

$$= \frac{1}{3}(27.588+j26.516) = 9.196+j8.84A$$

$$\overrightarrow{I_{a2}} = \frac{1}{3}(\overrightarrow{I_a}+a^2\overrightarrow{I_b}+a\overrightarrow{I_c})$$

$$= \frac{1}{3}((10+j4)+(-0.5-j0.866)(11-j9)+(-0.5+j0.866)(-15+j9))$$

$$= \frac{1}{3}(10+j4-13.294-j5.026-0.294-j17.49)$$

$$= \frac{1}{3}(-3.588 - j18.516) = -1.196 - j6.172A$$

1. Ans: b – phase

$$\overrightarrow{I_{b0}} = \overrightarrow{I_{a0}} = (2 + j1.33)A$$

$$\overrightarrow{I_{b1}} = a^2\overrightarrow{I_{a1}} = (-0.5 - j0.866)(9.196 + j8.84)$$
$$= 3.06 - j12.38A$$
$$\overrightarrow{I_{b2}} = a\overrightarrow{I_{a2}} = (-0.5 + j0.866)(-1.196 - j6.172)$$
$$= 5.94 + j2.05A$$

c – phase

$$\overrightarrow{I_{c0}} = \overrightarrow{I_{a0}} = (2 + j1.33)A$$

$$\overrightarrow{I_{c1}} = a\overrightarrow{I_{a1}} = (-0.5 + j0.866)(9.196 + j8.84)$$
$$= -12.25 + j3.54A$$
$$\overrightarrow{I_{c2}} = a^2\overrightarrow{I_{a2}} = (-0.5 - j0.866)(-1.196 - j6.172)$$
$$= -4.747 + j4.12A$$

2. The zero, positive and negative sequence voltages of phase 'a' are given below. Find out the phase voltages $\overrightarrow{V_a}$, $\overrightarrow{V_b}$

and $\overrightarrow{V_c}$.

$$\overrightarrow{V_0} = 200\angle 0^\circ, \; \overrightarrow{V_1} = 210\angle -30^\circ, \; \overrightarrow{V_2} = 150\angle 190^\circ$$

Ans:

$$\overrightarrow{V_a} = \overrightarrow{V_0} + \overrightarrow{V_1} + \overrightarrow{V_2}$$

$$= 200\angle 0 + 210\angle -30 + 150\angle 190$$

$$= 200 + 182 - j105 + -147.7 - j26.1$$

$$= 234.3 - j131.1 = 268.5\angle -29.2^\circ V$$
$$\overrightarrow{V_b} = \overrightarrow{V_0} + a^2\overrightarrow{V_1} + a\overrightarrow{V_2}$$
$$= 200\angle 0 + 1\angle 240 \times 210\angle -30 + 1\angle 120 \times 150\angle 190^\circ$$

$$= 200\angle 0 + 210\angle 210 + 150\angle 310$$

$$= 200 - 181.8 - j105 + 96.4 - j114.9$$

2. Ans:

$$= 114.6 - j219.9$$

$$= 248\angle -62.5^\circ V$$
$$\overrightarrow{V_c} = \overrightarrow{V_0} + a\overrightarrow{V_1} + a^2\overrightarrow{V_2}$$
$$= 200\angle 0 + 1\angle 120 \times 210\angle -30 + 1\angle 240 \times 150\angle 190$$

$$= 200\angle 0 + 210\angle 90 + 150\angle 70$$

$$= 200 + j210 + 51.3 + j141 = 251.3 + j351 = 431.7\angle 54.4 V$$

3. A 20MVA, 6.6kV 3-phase generator has a positive sequence impedance of j1.5 Ω, negative sequence impedance of

j1.0 Ω and zero sequence impedance of j0.5 Ω. and P_m = 0 (a) If a single phase to ground fault occurs on phase 'a' find out the fault current. (b) If the fault is through an impedance of j2 Ω, what will be the fault current?

Ans: The fault has occurred on 'a' phase. Taking 'a' phase as reference,

(a) $V_a = \dfrac{6.6\times 10^3}{\sqrt{3}} = 3810V$

For a single line to ground fault,

$$I_1 = I_2 = I_0 = \frac{V}{Z_1 + Z_2 + Z_0} = \frac{3810}{j1.5 + j1.0 + j0.5} = \frac{3810}{j3} = -j1270.2A$$

Fault current $I_{aF} = I_1 + I_2 + I_0 = 3I_1 = 3\times -j1270.2 = -j3810.5A$

(b) If the fault is through an impedance of j2 Ω

$$I_1 = I_2 = I_0 = \frac{V}{Z_1 + Z_2 + Z_0 + 3Z_f}$$

$$= \frac{3810}{j1.5 + j1.0 + j0.5 + (j2)\times 3}$$

$$= \frac{3810}{j9} = -j423.3A$$

$$I_{af} = 3I_1$$

4. In a 3ϕ system, if the per unit values of positive, negative and zero sequence reactances are given by $j0.1$, $j0.085$

and $j0.05$ respectively. Determine the fault current, if the fault is (a) L-L-G (b) L-L.

Ans: (a) For L-L-G fault involving phases b & c.

$$\overrightarrow{V_b} = \overrightarrow{V_c} = 0 \quad \overrightarrow{I_a} = 0, \quad Z_1 = j0.1pu \quad Z_2 = j0.085pu \quad Z_0 = j0.05pu$$

$$\overrightarrow{I_a} = \overrightarrow{I_{a0}} + \overrightarrow{I_{a1}} + \overrightarrow{I_{a2}} = 0$$

$$\overrightarrow{I_{a1}} = \frac{V}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

Let V = 1pu

i.e., $I_{a1} = \dfrac{1}{0.1j + \dfrac{j0.085\times j0.05}{j0.085 + j0.05}}$

$$= \frac{1}{j0.1 + j0.032} = \frac{1}{j0.132} = -j7.6pu$$

$$\overrightarrow{I_{a0}} = -I_{a1}\frac{Z_2}{Z_2+Z_0} = \frac{-(-j7.6)\times j0.05}{j0.085+j0.05}$$

$$= \frac{j7.6\times j0.05}{j0.135} = j2.82pu$$

$$\overrightarrow{I_{a0}} = \frac{1}{3}(\overrightarrow{I_a}+\overrightarrow{I_b}+\overrightarrow{I_c})$$

$= \frac{1}{3}(\overrightarrow{I_b}+\overrightarrow{I_c})$ since $\overrightarrow{I_a} = 0$ or $\overrightarrow{I_b}+\overrightarrow{I_c} = 3\overrightarrow{I_{a0}}$

i.e., Fault current $= \overrightarrow{I_b}+\overrightarrow{I_c} = 3\overrightarrow{I_{a0}}$

$$= 3\overrightarrow{I_{a0}} = 3\times j2.82pu$$

$$= j8.44pu$$

4. Ans: (b) L-L fault

For line to line fault between ‘b' and ‘c'

$$I_0 = 0$$

$$I_1 = -I_2 = \frac{V}{Z_1+Z_2}$$

$$I_1 = \frac{V}{Z_1+Z_2} = \frac{1}{j0.1+j0.085} = \frac{1}{j0.185} = -j5.4pu$$

$$I_2 = -(-j5.4) = j5.4$$

Fault current $= I_b = -I_c$

$$I_b = \overrightarrow{I_0}+a^2\overrightarrow{I_1}+a\overrightarrow{I_2}$$

i.e., $I_b = 0+(-0.5-j0.866)(-j5.4)+(-0.5+j0.866)(j5.4)$

$$= j2.7-4.68+-j2.7-4.68$$

$$= -9.36pu$$

i.e., Fault current = -9.36pu

5. Calculate the positive, negative and zero sequence impedance of a feeder if its self impedance is j1.67 Ω and mutual impedance is j0.67 Ω.

Self impedance $Z_S = 1.67\Omega$, mutual impedance $Z_m = 0.67\Omega$

Ans: Positive sequence impedance $= Z_S - Z_m$

= 1.67 – 0.67

= 1 Ω

Negative sequence impedance $= Z_S - Z_m$

= 1.67 – 0.67

= 1 Ω

Zero sequence impedance $= Z_S + 2Z_m$

$$= 1.67 + 2\times 0.67$$

= 3.01 Ω

6. Assuming b – phase to be reference phasor define the sequence transformation matrix.

Ans: With ‘b' phase as reference phasor, the transformation matrix can be defined as follows.

$$\begin{bmatrix} V_b \\ V_c \\ V_a \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{b0} \\ V_{b1} \\ V_{b2} \end{bmatrix}$$

Justifications:

Now, if $V_{b1} = V_{b2} = 0$, i.e. only zero sequence excitation is present, then we get

$\overrightarrow{V_b} = \overrightarrow{V_c} = \overrightarrow{V_a} = \overrightarrow{V_{b0}}$, thus we see that all the zero sequence components are extracted.

If $V_{b2} = V_{b0} = 0$ i.e., only positive sequence excitation is present, then,

$V_b = V_{b1}$[$\because$ V_b being reference phasor]

$V_c = a^2 V_{b1}$ [i.e., V_c lags V_b by $120°$]

$V_a = a V_{b1}$ [i.e., V_b lags V_a by $120°$]

Thus, the positive sequence component is properly extracted. Similarly, if

$V_{b1} = V_{b0} = 0$, only negative sequence excitation is present.

i.e., we will get

$V_b = V_{b2}$

$V_c = a V_{b2}$ [i.e., V_b lags V_c by $120°$]

$V_a = a^2 V_{b1}$ [i.e., V_a lags V_b by $120°$]

7. Comment if the two – sequence transformations obtained by taking ‘a' phase and ‘b' phase as reference are identical or

not.

Ans: With ‘a' phase as reference phasor, the sequence transformation is defined as,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \qquad (1)$$

or $V_{abc} = T_a V_a^{012}$

With ‘b' phase as reference phasor, the sequence transformation is defined as,

$$\begin{bmatrix} V_b \\ V_c \\ V_a \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \qquad (2)$$

Now, rearranging the equation (2) to follow the same order as (1) we get,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & a & a^2 \\ 1 & 1 & 1 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix}$$

or $V_{abc} = T_b V_b^{012}$

Clearly, T_a and T_b are not identical.

8. In problem No. 2 if the data represented sequence components with ‘b' phase as reference phasor, instead of ‘a'

phase, compute $\vec{V_a}$, $\vec{V_b}$ and $\vec{V_c}$. Comment on the result.

Ans: With ‘b' phase as reference phasor, the sequence transformation is given by,

$$\begin{bmatrix} V_b \\ V_c \\ V_a \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 200\angle 0 \\ 210\angle -30 \\ 150\angle 190 \end{bmatrix}$$

We will get $V_b = 268.5 \angle -29.2° = (V_a^{(old)})$

$$V_c = 248 \angle -62.5° (V_b^{(old)})$$

$$V_a = 431.7 \angle 54.4° (V_c^{(old)})$$

Hence, we can conclude that changing of reference phasor causes renaming of phasors and hence a different result.

9. Analyze a bolted S-L-G fault on phase ‘b' of an unloaded transmission line using sequence components with b – phase

as reference phasor.

Ans: With b- phase as reference phasor we have

$$V_b = V_{b0} + V_{b1} + V_{b2}$$

Now, for a bolted S-L-G fault $V_b^f = 0$;

Therefore,

$$\begin{bmatrix} I_{b0} \\ I_{b1} \\ I_{b2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_b \\ I_c \\ I_a \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_f \\ 0 \\ 0 \end{bmatrix}$$

i.e., $I_{b0} = I_{b1} = I_{b2} = I_f/3$

Based on 3 phase model of balanced circuit

$$\begin{bmatrix} \Delta V_b \\ \Delta V_c \\ \Delta V_a \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_b \\ I_c \\ I_a \end{bmatrix}$$

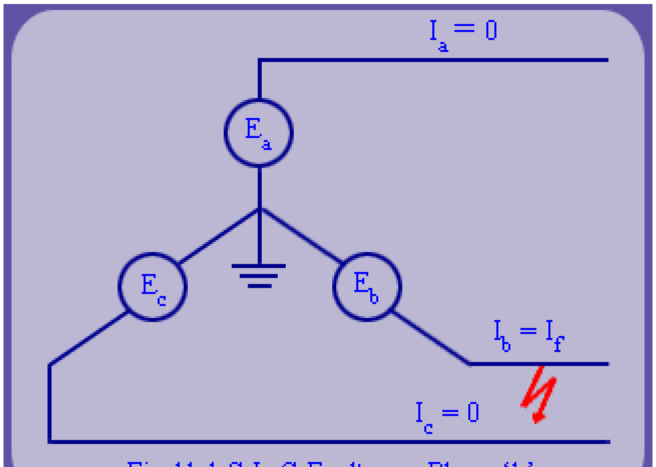

Fig 11.1 S-L-G Fault on a Phase ‘b'

Applying sequence transformation,

$$\begin{bmatrix} \Delta V_{b0} \\ \Delta V_{b1} \\ \Delta V_{b2} \end{bmatrix} = T^{-1}\ Z\ T \begin{bmatrix} I_{b0} \\ I_{b1} \\ I_{b2} \end{bmatrix}$$

$$\Delta V_b^{012} = diag(Z_0\, Z_1\, Z_2) I_b^{012}$$

or

$$\Delta V_{b0} = Z_0\, I_{b0}$$

$$\Delta V_{b1} = Z_1 I_{b1}$$
$$\Delta V_{b2} = Z_2 I_{b2}$$
where $Z_0 = Z_s + 2Z_m$
$$Z_1 = Z_2 = Z_s - Z_m$$

9. Ans: The terminal voltages are given by,

$$\begin{bmatrix} V_b \\ V_c \\ V_a \end{bmatrix} = \begin{bmatrix} E_b \\ E_c \\ E_a \end{bmatrix} - \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_b \\ I_c \\ I_a \end{bmatrix}$$

Applying sequence transformation with b – phase as reference phasor,

$$\begin{bmatrix} V_{b0} \\ V_{b1} \\ V_{b2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_b \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & & \\ & Z_1 & \\ & & Z_2 \end{bmatrix} \begin{bmatrix} I_{b0} \\ I_{b1} \\ I_{b2} \end{bmatrix}$$

Now for a bolted fault on b - phase,

$$V_b = 0$$

i.e., $V_{b0} + V_{b1} + V_{b2} = 0$

$$E_b - (Z_0 + Z_1 + Z_2) I_{b0} = 0$$

or $I_{b0} = \dfrac{E_b}{Z_0 + Z_1 + Z_2}$

Thus, to analyze S-L-G fault on b - phase or a - c L-L fault or L-L-G fault we should take b – phase as reference phasor in sequence computation.

10. Derive the relationship between zero, positive and negative sequence phasors defined with ‘b' as reference phasor and
corresponding sequence phasors defined with ‘a' as reference phasor.

Ans: With ‘a' as reference phasor, the sequence transformation is defined as,

$$\begin{bmatrix} I_0^a \\ I_1^a \\ I_2^a \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

With ‘b' as reference phasor,

$$\begin{bmatrix} I_0^b \\ I_1^b \\ I_2^b \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_b \\ I_c \\ I_a \end{bmatrix}$$

10. Ans: For zero sequence phasor,

$$I_0^a = \frac{1}{3}(I_a + I_b + I_c)$$
$$I_0^b = \frac{1}{3}(I_b + I_c + I_a)$$

Therefore, $I_0^b = I_0^a$

Positive sequence phasor,

$$I_1^a = \frac{1}{3}(I_a + aI_b + a^2 I_c)$$

$$I_1^b = \frac{1}{3}(I_b + aI_c + a^2 I_a)$$

$$= \frac{1}{3} \times \frac{a}{a}(I_b + aI_c + a^2 I_a)$$

$$= \frac{1}{3a}(aI_b + a^2 I_c + a^3 I_a)$$

Since $a^3 = 1$,

$$I_1^b = \frac{1}{3a}(I_a + aI_b + a^2 I_c)$$

$$= \frac{1}{a} \times \frac{1}{3}(I_a + aI_b + a^2 I_c) = \frac{1}{a} \times I_1^a$$

or, $I_1^a = aI_1^b$

i.e., positive sequence current with ‘b' as reference phasor lags by $120°$ with positive sequence current with ‘a' as reference phasor.

Negative sequence phasor,

$$I_2^a = \frac{1}{3}(I_a + a^2 I_b + aI_c)$$

$$I_2^b = \frac{1}{3}(I_b + a^2 I_c + aI_a)$$

$$= \frac{1}{3}(a^3 I_b + a^2 I_c + aI_a)$$

$$= \frac{1}{3} \times a(a^2 I_b + aI_c + I_a)$$

$$= a \times \frac{1}{3}(I_a + a^2 I_b + aI_c)$$

$$= aI_2^a$$

i.e., negative sequence current with ‘b' as reference phasor leads the negative sequence current with ‘a' as reference phasor, by $120°$.

TOP

Review Questions

1. Derive the relationship between the transformation matrices T_a and T_c with 'a' and 'c' as reference phasors respectively.
2. Derive the relationship between positive, negative and zero sequence phasors with 'c' as reference phasor with

   corresponding sequence phasor with 'b' as reference phasor.
3. Out of the four fault types (S-L-G, L-L, L-L-G and 3ϕ) magnitude of which fault current will be the highest and why?
4. Find the symmetrical components if $V_a = 200\angle 30°$, $V_b = 180\angle -60°$ and $V_c = 150\angle 145°$.
5. The zero, positive and negative currents of phase ‘a' are given by (5+j1)A, (7.5 – j1.2)A and (6+j2)A respectively. Find out.

   $\vec{I_a}$, $\vec{I_b}$ and $\vec{I_c}$.
6. A 3ϕ, 20MVA, 11kV generator with positive, negative and zero sequence impedance j2Ω, j1.8Ω and j0.6Ω is

connected to a feeder with sequence impedance j1.5Ω, j1.5Ω and j4.5Ω. If a S-L-G fault occurs at the remote end of the feeder, calculate the fault current.

7. Find out the ratio of fault currents for S-L-G fault to bolted 3ϕ fault of a generator with $Z_1 = j1.0pu$, $Z_2 = j0.8pu$ and $Z_0 = j0.3pu$. Comment on your findings.

8. In a 3ϕ system, the pu values of positive, negative and zero sequence impedances are given by j1.5, j1.25 and j0.6 respectively. The fault impedance is given by j1Ω. Determine the fault current for L-L fault and L-L-G fault.

Recap

In this lecture we have learnt the following:

- To calculate sequence components for an unbalanced set of phasors.
- To find out the unbalanced phasors from a given set of sequence components.
- Relationship between sequence transformation matrices with 'b' and 'c' as reference phasors.
- To find out fault currents for different types of faults.
- To calculate the sequence impedance of a feeder.