EE 102 **Homework #2 Solutions**

Fall 2001 **Professor Paganini**

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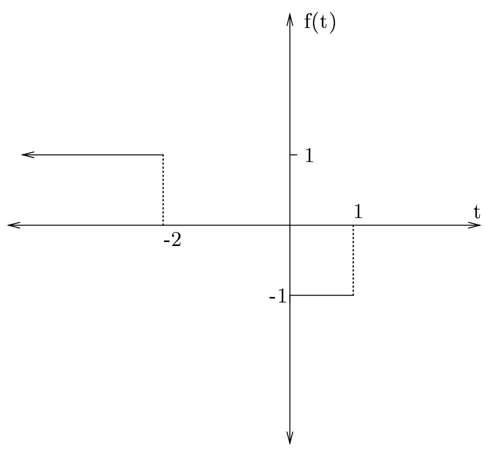
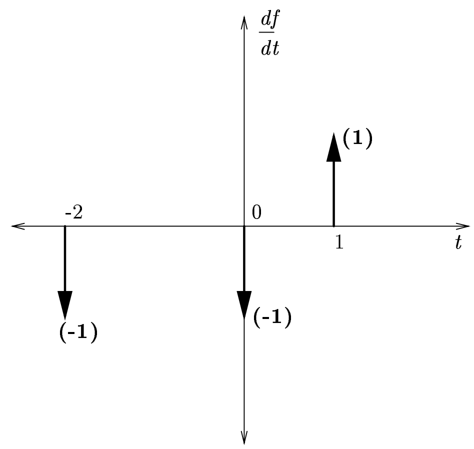
1. (a) $f(t) = 1 - u(t+2) - u(t) + u(t-1)$.

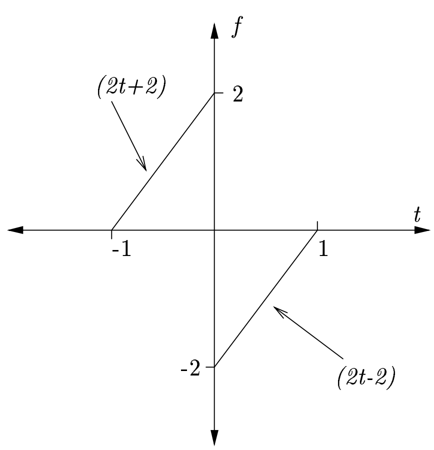
   $\frac{df}{dt}(t) = -\delta(t+2) - \delta(t) + \delta(t-1)$.

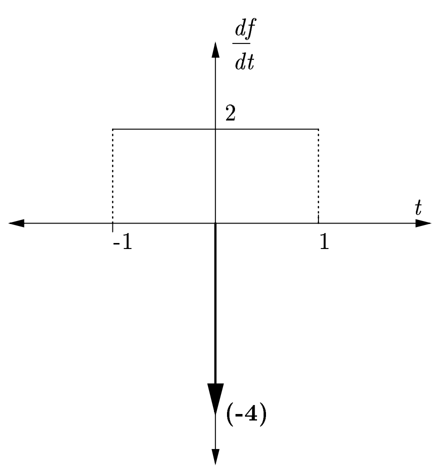




   (b) $f(t) = (2t+2)[u(t+1) - u(t)] + (2t-2)[u(t) - u(t-1)]$.

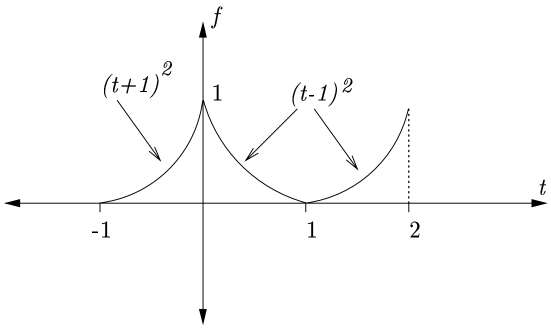
   $\frac{df}{dt}(t) = 2[u(t+1) - u(t-1)] - 4\delta(t)$.

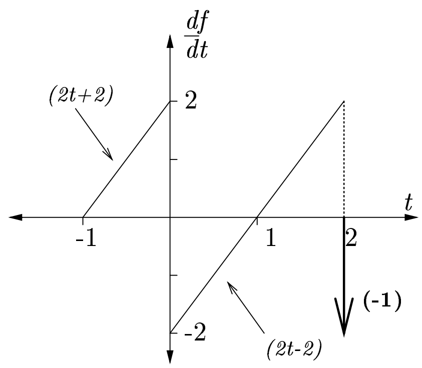




(c) $f(t) = (t+1)^2[u(t+1) - u(t)] + (t-1)^2[u(t) - u(t-2)]$.

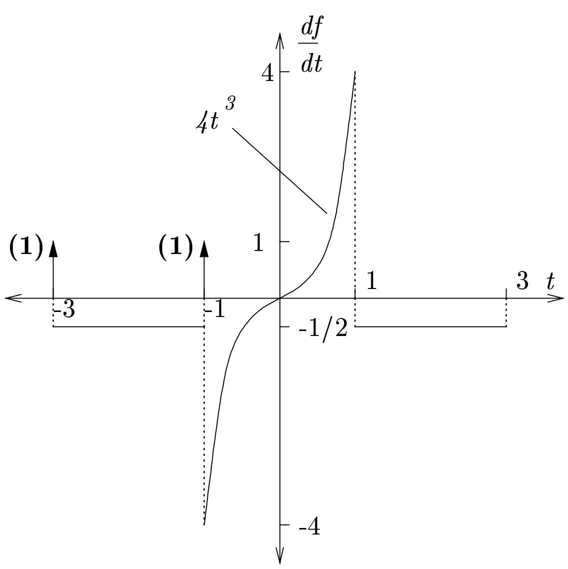
$\frac{df}{dt}(t) = 2(t+1)[u(t+1) - u(t)] + 2(t-1)[u(t) - u(t-2)] - \delta(t-2)$.





2. (a)

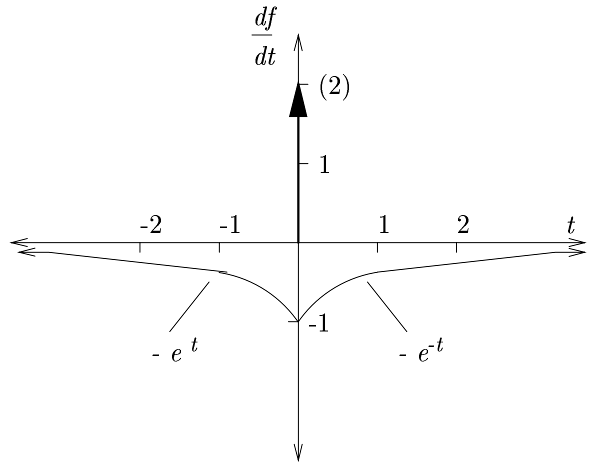
$$\begin{aligned} f(t) &= (-\frac{t}{2} - \frac{1}{2})[u(t+3) - u(t+1)] + t^4[u(t+1) - u(t-1)] \\ &\quad +(-\frac{t}{2} + \frac{3}{2})[u(t-1) - u(t-3)]. \\ \frac{df}{dt}(t) &= \delta(t+3) - \frac{1}{2}[u(t+3) - u(t+1)] + \delta(t+1) + 4t^3[u(t+1) - u(t-1)] \\ &\quad +(-\frac{1}{2})[u(t-1) - u(t-3)]. \end{aligned}$$



(b)

$f(t) = -e^t u(-t) + e^{-t} u(t).$

$\frac{df}{dt}(t) = -e^t u(-t) + 2\delta(t) - e^{-t} u(t).$



3. (a) $\int_{-\infty}^{\infty} e^{\sin(\pi t)}\ \delta(t+\frac{1}{2})\ dt = e^{\sin(\pi(-\frac{1}{2}))} = e^{-1} = 1/e$

   (b) $\int_{-\infty}^{3} e^{t^2-3t-4}\delta(t-4)\ dt = 0$ since $4 \notin (-\infty, 3]$.

   (c) $\int_{a-}^{\infty} \cos(t)\delta(t-a)\ dt = \cos(a)$ since the impulse at $t = a$ is included in the domain of integration.

4. (a)

$$y(t) = T[x(t)] = \int_{-\infty}^{t} \cos(t+\sigma)x(\sigma-1)d\sigma$$

   By definition,

$$h(t,\tau) = T[\delta(t-\tau)] = \int_{-\infty}^{t} \cos(t+\sigma)\delta(\sigma-\tau-1)d\sigma = \cos(t+\tau+1)u(t-\tau-1)$$

   (b) The system is not time-invariant since $h(t,\tau)$ is not a function of $(t-\tau)$ alone.
   The system is causal since $h(t,\tau) = 0$ for $(t-\tau) < 0$.

5. As solved in Homework # 1,

$$y(t) = T[x(t)] = x(t) - 3\int_{0-}^{t} e^{-(t-\sigma)}x(\sigma)d\sigma, \quad t \geq 0$$

   This system is linear and time-invariant.

   By definition,

$$h(t) = T[\delta(t)] = \delta(t) - 3\int_{0-}^{t} e^{-(t-\sigma)}\delta(\sigma)d\sigma = \delta(t) - 3e^{-t}u(t).$$