Electric Power

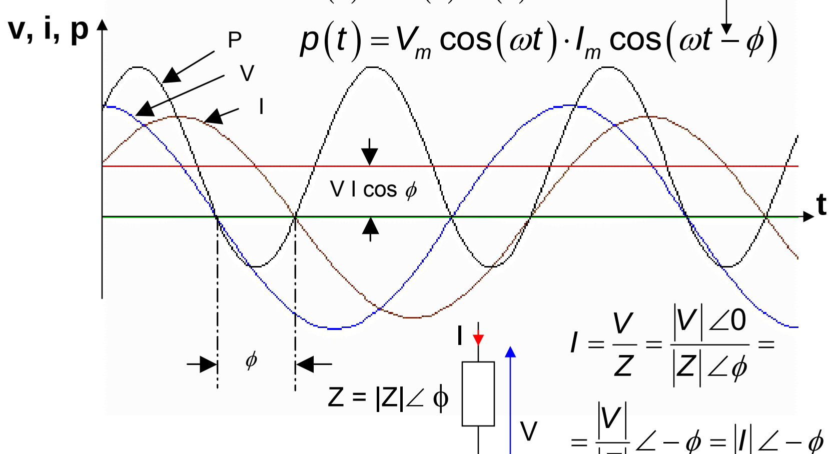
- **single-phase (1~) systems**
  - One source
  - Two energy-carrying wires
  - Pulsating instantaneous power (2f)
- **three-phase (3~) systems**
  - Three sources, 120° phase shifted

Single Phase Power

Definition: $p(t) = v(t) \cdot i(t)$

Lagging, for ϕ > 0

$$p(t) = V_m \cos(\omega t) \cdot I_m \cos(\omega t - \phi)$$



$$I = \frac{V}{Z} = \frac{|V|\angle 0}{|Z|\angle \phi} = \frac{|V|}{|Z|}\angle -\phi = |I|\angle -\phi$$

Instantaneous and Average Power

trigonometric identity:

$$\cos\alpha \cdot \cos\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

the instantaneous power equation:

$$p(t)=\frac{V_m\, I_m}{2}\left[\cos\phi+\cos(2\omega t-\phi)\right]$$

the average of $\cos(2\omega t-\phi)$ over a cycle is zero

the average power over one cycle: $P=\tfrac{1}{2}V_m\, I_m\cos(\phi)$

Average Power from RMS Values

$$V_{rms}=\frac{1}{T}\sqrt{\int_0^T v^2(t)\mathrm{d}t}=\frac{1}{\sqrt{2}}V_m \quad \textbf{for } v(t)=V_m\sin(\omega t)$$

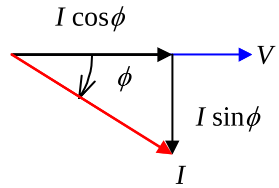
Root mean square (rms) of an arbitrarily (periodic) AC waveform delivers the same power to a resistor as the corresponding DC quantity.

The average power in terms of *rms*: $P=V_{rms}I_{rms}\cos(\phi)$

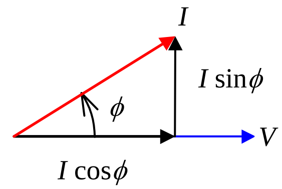
Notation convention (this lecture, not in general):
Voltage and currents given in rms if not stated otherwise.
e.g. 208 V ∠ -30° means V_m = √2·208 = 294 V_{pk}

Power Factor

$$P = V_{rms}\, I_{rms} \underbrace{\cos(\phi)}_{\text{power factor}}$$



Lagging Power Factor



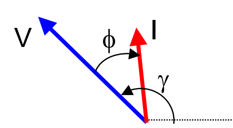
Leading Power Factor

Load behaves like	**R || jωL**	**R || 1/jωC**

Complex Power

General case: $V = |V|\angle\gamma$

$I = |I|\angle\gamma - \phi$



Complex or Apparent Power: $S = V \cdot I^* = |V|\cdot|I|\angle\phi$

$S = |V|\cdot|I|\cos\phi + j|V|\cdot|I|\sin\phi$

Real Power: $P = |V|\cdot|I|\cos(\phi) \qquad (W)$

Reactive Power: $Q = |V|\cdot|I|\sin(\phi) \qquad (VAr)$

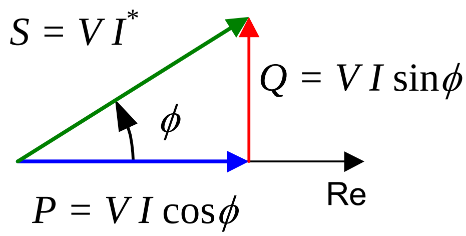
$$\boxed{S = P + jQ \qquad (VA)}$$

Positive Angle ϕ: Lagging PF

Negative Angle ϕ: Leading PF

Power Triangle

Power Factor: $\cos\phi = \cos\left[\arctan\left(\frac{Q}{P}\right)\right] = \frac{P}{\sqrt{P^2+Q^2}} = \frac{P}{|S|}$



+ Q = Lagging Power Factor
Inductive Load (Consuming)

- Q = Leading Power Factor
Capacitive Load (Consuming)

Example: Power Factor

See Book, Problem 1-18, p. 54

Assume that the voltage applied to a load is V = 208 V ∠− 30° and the current flowing through the load is I = 5 A ∠ 15°.
(a) Calculate the complex power S consumed by this load.
(b) Is this load inductive or capacitive?
(c) Calculate the power factor of this load?
(d) Calculate the reactive power consumed or supplied by this load. Does the load consume reactive power from the source or supply it to the source?

Example: Power Factor

(a) The complex power S consumed by this load is

$$\mathbf{S} = \mathbf{VI}^* = (208\angle -30^\circ \text{ V})(5\angle 15^\circ \text{ A})^* = (208\angle -30^\circ \text{ V})(5\angle -15^\circ \text{ A})$$

$$\mathbf{S} = 1040\angle -45^\circ \text{ VA}$$

(b) This is a capacitive load.

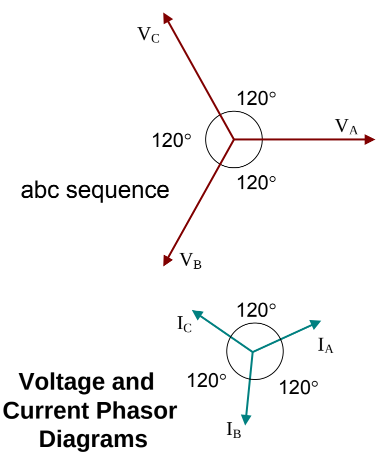
(c) The power factor of this load is

$$\text{PF} = \cos(-45^\circ) = 0.707 \text{ leading}$$

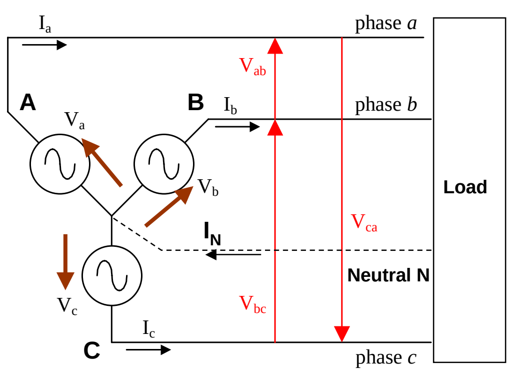
(d) This load supplies reactive power to the source. The reactive power of the load is

$$Q = VI\sin\theta = (208 \text{ V})(5 \text{ A})\sin(-45^\circ) = -735 \text{ var}$$

Wye (Y) Connected Systems



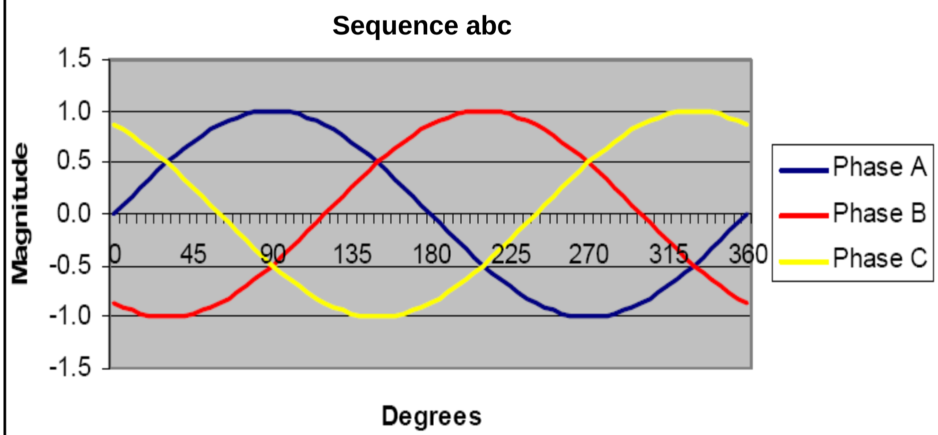
Voltage and Current Phasor Diagrams



Wiring Connection

3-wire system: $I_N = 0$, Balanced system: $I_a = I_b = I_c$

Three Phase Time Domain Signals



Three-Phase Power, <u>Balanced</u> System

$$V_a(t) = \sqrt{2} \cdot V_p \sin(\omega t) \qquad I_a(t) = \sqrt{2} \cdot I_p \sin(\omega t - \phi)$$
$$V_b(t) = \sqrt{2} \cdot V_p \sin(\omega t - 120°) \qquad I_b(t) = \sqrt{2} \cdot I_p \sin(\omega t - 120° - \phi)$$
$$V_c(t) = \sqrt{2} \cdot V_p \sin(\omega t + 120°) \qquad I_c(t) = \sqrt{2} \cdot I_p \sin(\omega t + 120° - \phi)$$

$$p_a(t) = V_a(t) \cdot I_a(t) = 2V_p I_p \sin(\omega t)\sin(\omega t - \phi)$$
$$p_b(t) = V_b(t) \cdot I_b(t) = 2V_p I_p \sin(\omega t - 120°)\sin(\omega t - 120° - \phi)$$
$$p_c(t) = V_c(t) \cdot I_c(t) = 2V_p I_p \sin(\omega t + 120°)\sin(\omega t + 120° - \phi)$$

Three-Phase Power, Balanced System

$$p_{3\phi}(t) = p_a(t) + p_b(t) + p_c(t)$$

$$\sin\alpha \cdot \sin\beta = \tfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$p_{3\phi}(t) = V_p I_p \left\{3\cos\phi - \underbrace{\left[\cos(2\omega t-\phi) + \cos(2\omega t - 240 - \phi) + \cos(2\omega t + 240 - \phi)\right]}_{=0}\right\}$$

$$\boxed{p_{3\phi}(t) = 3 \cdot V_p I_p \cos(\phi) = P_{3\phi} = const.!}$$

Complex Power, Balanced System

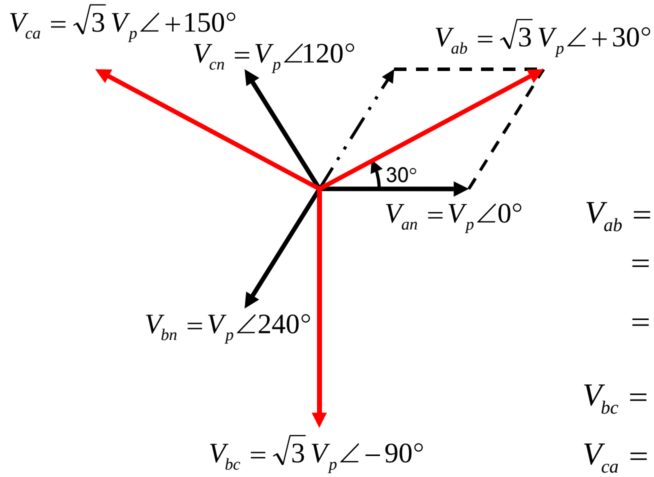
$$V_p = |V_p| \angle 0^\circ \qquad I_p = |I_p| \angle -\phi$$

$$S_{3\phi} = 3 \cdot V_p \cdot I_p^* = \sqrt{3} \cdot V_L \cdot I_L^*$$ **General Form of Complex Power**

$$S_{3\phi} = 3|V_p||I_p| \angle \phi$$ **Phasor Form**

$$S_{3\phi} = 3|V_p||I_p|(\cos\phi + j\sin\phi) = P_{3\phi} + jQ_{3\phi}$$

Relations in the Wye Connection



$$V_{ab} = V_{an} - V_{bn}$$
$$= V_p\angle 0° - V_p\angle -120°$$
$$= \sqrt{3}\cdot V_p\angle 30°$$

$$V_{bc} = \sqrt{3}\cdot V_p\angle -90°$$
$$V_{ca} = \sqrt{3}\cdot V_p\angle 150°$$

Relations in the Y Connection

voltage relationship

$$V_{Line} = \sqrt{3}\cdot V_{phase}\angle +30°$$

current relationship

$$I_{Line} = I_{phase}$$

Three-Phase Power, Balanced System

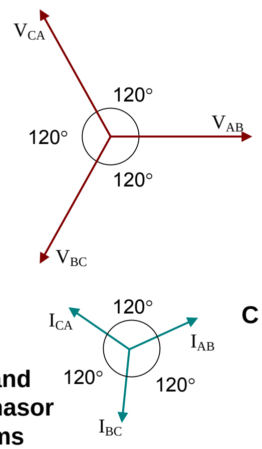
$$P_{3\phi} = 3|V_p||I_p|\cos\phi = \sqrt{3}|V_L||I_L|\cos\phi$$ **Real Power**

$$Q_{3\phi} = 3|V_p||I_p|\sin\phi = \sqrt{3}|V_L||I_L|\sin\phi$$ **Reactive Power**

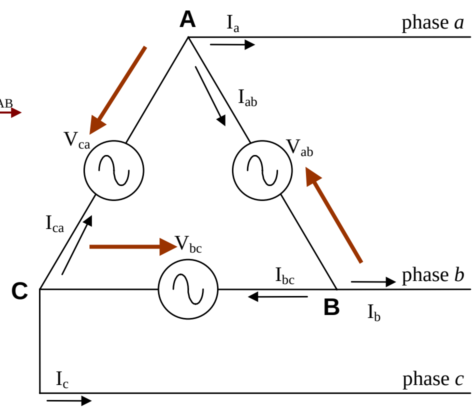
Electric Power

- **single-phase (1~) systems**
  - One source
  - Two energy-carrying wires
  - Pulsating instantaneous power (2f)
- **three-phase (3~) systems**
  - Three sources, 120° phase shifted
  - Requires only 3 energy-carrying wires to deliver 3 times the power of 1~ phase system
  - Balanced systems
    - Instantaneous power is **constant**
    - No torque pulsations in 3~ machines
  - Unbalanced systems
    - Instantaneous power is not constant
    - Analyze each phase individually as a 1~ system

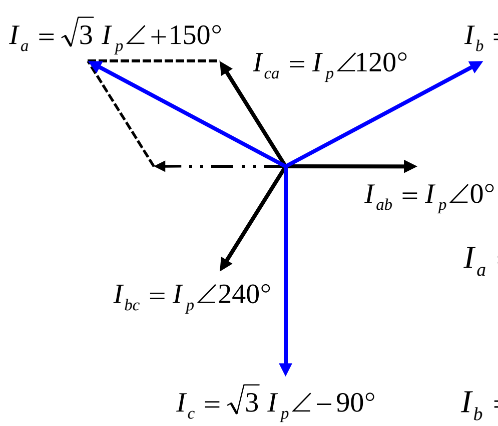
Relations in the Delta Connection



Voltage and Current Phasor Diagrams



Relations in the Delta (Δ) Connection



$$I_a = I_{ca} - I_{ab} = I_p \angle 120° - I_p \angle 0°$$
$$= \sqrt{3} \cdot I_p \angle 150°$$

$$I_b = \sqrt{3} \cdot I_p \angle 30°$$
$$I_c = \sqrt{3} \cdot I_p \angle -90°$$

Relations in the Delta Connection

voltage relationship

$$V_{Line} = V_{phase}$$

current relationship

$$I_{Line} = \sqrt{3} \cdot I_{phase} \angle +30^{\circ}$$

See also Book,
Table 2-1

Three-Phase Power, Balanced System

Wye Connection

$$I_p = I_L$$

$$V_p = V_L / \sqrt{3}$$

$$P_{3\phi} = 3 \cdot V_p I_p \cos(\phi)$$

Delta Connection

$$I_p = I_L / \sqrt{3}$$

$$V_p = V_L$$

$$P_{3\phi} = 3 \cdot V_p I_p \cos(\phi)$$

$$P_{3\phi} = \sqrt{3} \cdot V_L I_L \cos(\phi)$$

Homework 1

See web site
http://www.eng.fsu.edu/~steurer/eel3216.html