# Transmission Lines

- Overhead Conductor
- Overhead Spacer Cable
- Underground Cable
- Three-Conductor Cable
- Service Cables

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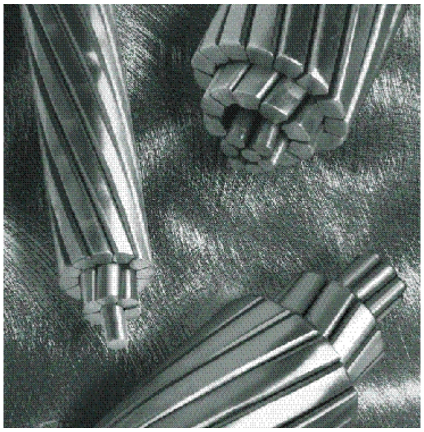
# Overhead Conductors

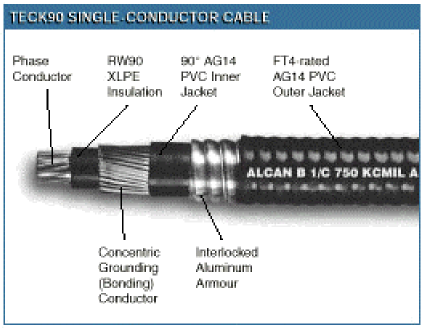
- ACSR
  Aluminum Conductor with inner Steel Reinforced strands
- ACAR
  Aluminum Conductor with inner Al-Alloy Reinforced strands (corrosion resistant)
- Aluminum - current carrying member
- Steel - structural support

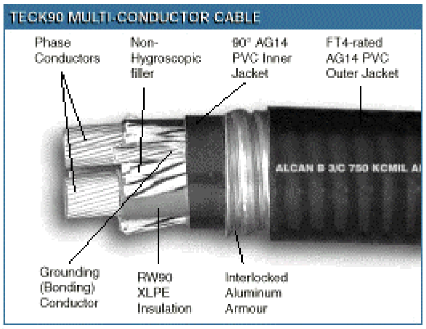


# Overhead Cable

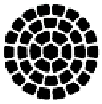
- **Where conductor close proximity is required**
- **Insulating jacket surrounds each conductor**
- **Plastic spacers keep conductors from coming in contact with one another**

# Cables





**Specification**
CSA C22.2 No. 131 (TECK)
CSA C22.2 No. 174 (Hazardous Locations)
FT4-Rated: Vertical Flame Test - cable in cable tray (IEEE P1202)
AG14 inner and outer PVC jacket:
Maximum 14% acid gas emission by weight
FMRC Class 3972 Fire Test GP-2 (jacketed)
GP-1 (unjacketed)



# Cables

- Underground transmission and distribution cables
- Semiconducting material surrounds the conductor to grade the electric field
- Plastic jacket provides insulation and protection
- Neutral strands for an outer shell for protection and return currents

# Transmission Line Parameters

- **Unlike transformers and generators, transmission lines do not have nameplate data by which to construct a model for analysis**
- **Modeling of transmission lines with:**
  - geometric data – conductor distances and diameters
  - wire data – resistance and surface area
- **Parameters derived from modeling**
  - Resistance
  - Self and mutual inductances
  - Capacitances between conductors and against ground

# Resistance

- **AWG – American Wire Gauge system for size**
  - HISTORICAL geometrical progression due to wire drawing mechanism
- **Solid round conductor**
  - AWG 36 → d = 0.005 in = 5 mil
  - Approximately doubles diameter every 6 gauges
  - Approximately doubles area every 3 gauges

$$d = 0.005 \cdot 96^{\frac{36-g}{39}}$$

- **Wire larger than AWG 4/0 (AWG “0000”, “four-ought”) measured in kcmil = 1000 circular mils**
  - One circular mil is the area of a circle with a diameter of one mil
- **Consult tables for resistance per length of any given conductor size/shape**
  - Large conductor sizes are often stranded for flexibility
    - geometric diameter is larger than diameter of the equivalent solid conductor

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# Resistance

- **DC resistance from length (l), cross section area (A), and material specific resistivity ρ**

$$R_{DC} = l\frac{\rho}{A}$$

- **In practice: resistance per length from tables**
- **Linear temperature dependency**

$$\rho(T) = \rho_{20°C}\left(1+\alpha\left(T-20°C\right)\right) \rightarrow M = \frac{1}{\alpha} \rightarrow \rho_{T2} = \rho_{T2}\frac{M+T_2}{M+T_1}$$

Resistivity and temperature constants of different materials

| **Material** | **Resistivity at 20°C (Ω-m)** | **Temperature constant (°C)** |
|---|---|---|
| Annealed copper | $1.72 \times 10^{-8}$ | 234.5 |
| Hard-drawn copper | $1.77 \times 10^{-8}$ | 241.5 |
| Aluminum | $2.83 \times 10^{-8}$ | 228.1 |
| Iron | $10.00 \times 10^{-8}$ | 180.0 |
| Silver | $1.59 \times 10^{-8}$ | 243.0 |

- **Others: Skin effect (50 Hz, 60 Hz, harmonics)**

# All Aluminum Conductor (AAC)

**1 inch = 25.4 mm (exact)**
**1 kcmil = 0.5067 mm$^2$**

| AWG | kcmil | Strands | Diameter, in. | GMR, ft | Resistance, Ω/1000 ft dc 20°C | Resistance, Ω/1000 ft 60-Hz ac 25°C | 60-Hz ac 50°C | 60-Hz ac 75°C | Breaking. Strength, lb | Weight, lb/1000 ft |
|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 26.24 | 7 | 0.184 | 0.0056 | 0.6593 | 0.6725 | 0.7392 | 0.8059 | 563 | 24.6 |
| 4 | 41.74 | 7 | 0.232 | 0.0070 | 0.4144 | 0.4227 | 0.4645 | 0.5064 | 881 | 39.1 |
| 2 | 66.36 | 7 | 0.292 | 0.0088 | 0.2602 | 0.2655 | 0.2929 | 0.3182 | 1350 | 62.2 |
| 1 | 83.69 | 7 | 0.328 | 0.0099 | 0.2066 | 0.2110 | 0.2318 | 0.2527 | 1640 | 78.4 |
| 1/0 | 105.6 | 7 | 0.368 | 0.0111 | 0.1638 | 0.1671 | 0.1837 | 0.2002 | 1990 | 98.9 |
| 2/0 | 133.1 | 7 | 0.414 | 0.0125 | 0.1299 | 0.1326 | 0.1456 | 0.1587 | 2510 | 124.8 |
| 3/0 | 167.8 | 7 | 0.464 | 0.0140 | 0.1031 | 0.1053 | 0.1157 | 0.1259 | 3040 | 157.2 |
| 4/0 | 211.6 | 7 | 0.522 | 0.0158 | 0.0817 | 0.0835 | 0.0917 | 0.1000 | 3830 | 198.4 |
|  | 250 | 7 | 0.567 | 0.0171 | 0.0691 | 0.0706 | 0.0777 | 0.0847 | 4520 | 234.4 |
|  | 250 | 19 | 0.574 | 0.0181 | 0.0693 | 0.0706 | 0.0777 | 0.0847 | 4660 | 234.3 |
|  | 266.8 | 7 | 0.586 | 0.0177 | 0.0647 | 0.0663 | 0.0727 | 0.0794 | 4830 | 250.2 |
|  | 266.8 | 19 | 0.593 | 0.0187 | 0.0648 | 0.0663 | 0.0727 | 0.0794 | 4970 | 250.1 |
|  | 300 | 19 | 0.629 | 0.0198 | 0.0575 | 0.0589 | 0.0648 | 0.0705 | 5480 | 281.4 |
|  | 336.4 | 19 | 0.666 | 0.0210 | 0.0513 | 0.0527 | 0.0578 | 0.0629 | 6150 | 315.5 |
|  | 350 | 19 | 0.679 | 0.0214 | 0.0494 | 0.0506 | 0.0557 | 0.0606 | 6390 | 327.9 |
|  | 397.5 | 19 | 0.724 | 0.0228 | 0.0435 | 0.0445 | 0.0489 | 0.0534 | 7110 | 372.9 |
|  | 450 | 19 | 0.769 | 0.0243 | 0.0384 | 0.0394 | 0.0434 | 0.0472 | 7890 | 421.8 |

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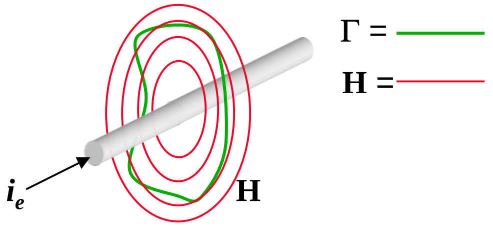
# Review of Magnetics

**Ampere's circuital law**

$$F = \oint_{\Gamma} \mathbf{H} \cdot d\,\mathbf{l} = i_e$$

**Integral of the scalar product of a closed path and the magnetic field equals the encircled current**

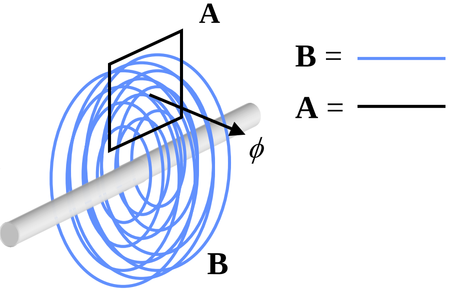


**Magnetic Flux**

$$\mathbf{B} = \mu \mathbf{H}$$

**Integral of the flux density that is normal to a defined area**

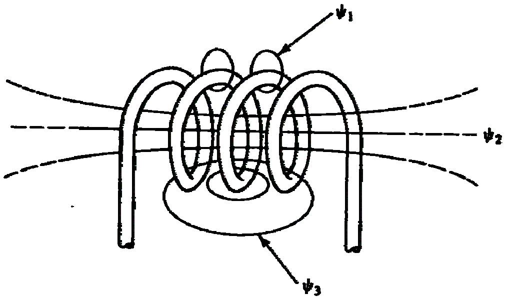
$$\phi = \int_A \mathbf{B} \cdot d\mathbf{a}$$



# Review of Magnetics

**Flux Linkage**

$$\lambda = \sum_{i=1}^{N} \phi_i$$



**is the total flux linked to the current**

**Inductance (definition)**

$$\lambda = Li$$

$$L = \frac{\lambda}{i} = \frac{\sum \phi}{i} = \frac{\sum \int \mathbf{B} \cdot d\mathbf{a}}{i} = \frac{\sum \int \mu \mathbf{H} \cdot d\mathbf{a}}{i}$$

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# Flux Linkage of Infinite Straight Wire

- **The infinite straight wire is an approximation of a reasonably long wire (a first step for a transmission line)**
- **Assumptions**
  - Image the wire to close the circuit at +/- infinity, establishing “one-turn coil” with the return path at infinity
  - Straight, infinitely long wire of radius $r$
  - Uniform current density in the wire. Total current is $i$
    - Valid in practice for dc or low-frequency ac
  - Flux lines form concentric circles (i.e. $\mathbf{H}$ is tangential)
  - Angular symmetry - it suffices to consider $H(x)$

# Flux Linkage of Infinite Straight Wire

- **Case 1: Points outside of the conductor ($x > r$)**

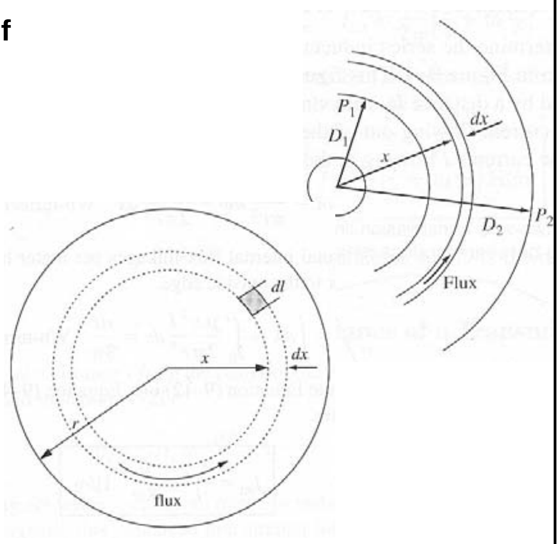
$$\oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = H \cdot 2\pi x = i$$

$$H = \frac{i}{2\pi x}$$

- **Case 2: Points inside of the conductor ($x < r$)**

$$H \cdot 2\pi x = i_e = \frac{\pi x^2}{\pi r^2} i$$

$$H = \frac{x}{2\pi r^2} i$$



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# Flux Linkage of Infinite Straight Wire

- **To find the flux density:**

$$B = \mu_r \mu_0 H$$

- **Outside the conductor**
  - Air $\mu_r \approx 1$
- **Inside the conductor**
  - Aluminum $\mu_r \approx 1$
  - Steel / Iron $\mu_r > 1$

typically on the order of 1000 to 10000

## Flux Linkage of Infinite Straight Wire

- **Case 1: Points outside of the conductor ($x > r$)**

$$\frac{\lambda_1}{l} = \frac{\phi_1}{l} = \frac{1}{l}\int_A \mathbf{B}\cdot d\mathbf{a} = \int_r^R B(x)\,dx = \mu_0 \int_r^R \frac{i}{2\pi x}\,dx$$

$$= \frac{\mu_0\, i}{2\pi}\ln\frac{R}{r}$$

- **Case 2: Points inside of the conductor ($x < r$)**

$$\frac{\lambda_2}{l} = \int_0^r \frac{\pi x^2}{\pi r^2}\underbrace{\mu_r\mu_0 H(x)\,dx}_{\frac{d\phi_2}{l}} = \mu_r\mu_0\int_0^r \frac{\pi x^2}{\pi r^2}\,\frac{x}{2\pi r^2}\,i\,dx$$

$$= \frac{\mu_r\mu_0\, i}{8\pi}$$

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## Flux Linkage of Infinite Straight Wire

**Combine the two flux linkages (per length) together**

$$\frac{\lambda}{l} = \frac{\lambda_1 + \lambda_2}{l} = \frac{\mu_0\, i}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{R}{r}\right)$$

$$= 2\times 10^{-7}\, i\left(\frac{\mu_r}{4} + \ln\frac{R}{r}\right)$$

**Wire inductance per length :**

$$\frac{L}{l} = \frac{\lambda}{li} = 2\times 10^{-7}\left(\frac{\mu_r}{4} + \ln\frac{R}{r}\right)$$

**Independent of conductor size!**

**R…defines area outside of conductor for which**

$$\oint_\Gamma \mathbf{H}\cdot d\mathbf{l} \neq 0$$

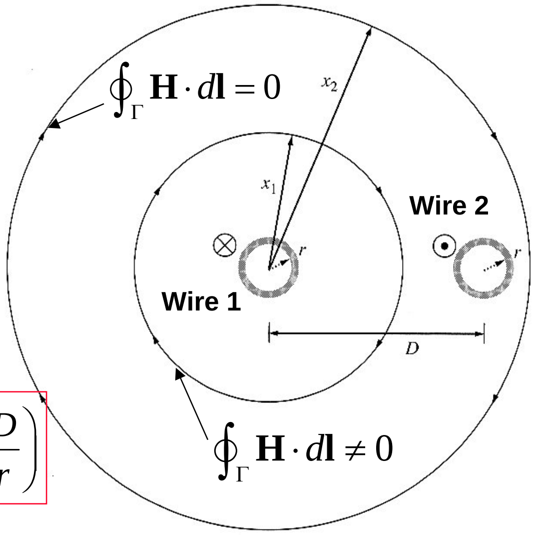
**r…conductor radius**

## Inductance per length of Two-Wire T-Line

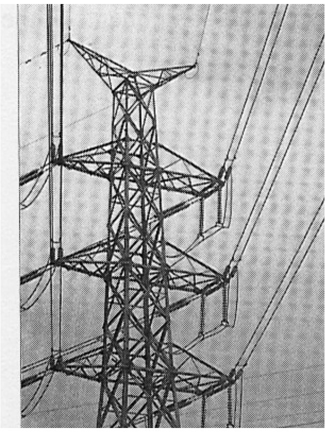
$\mu_r$ = 1 for non ferromagnetic conductors

$$L_1' = L_2' = \frac{\mu_0}{2\pi}\left(\frac{\mu_r}{4} + \ln\frac{D}{r}\right)$$

$$L_{12}' = L_1' + L_2' = \boxed{\frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right)}$$

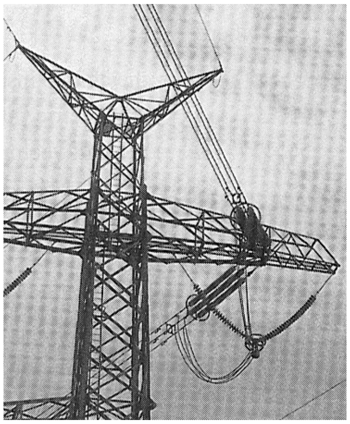


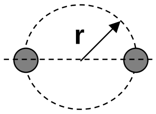
## Bundled Conductors



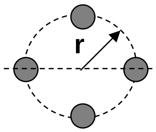
**Bundles reduce the electric field strength in the vicinity of the conductor → higher voltages possible w/o extensive corona losses (“sparking”)**

**Also: reduces L’**





**r’…equivalent radius for L’ calculation**



## Line Reactance

- **Reactance per length from L'**
  - e.g. single-phase (2 conductor) T-line

$$X_{12}^{'} = 2\pi f\, L_{12}^{'} \quad in \frac{\Omega}{m}$$

- **Large spacing between phases increases X'**
  - High voltage lines tend towards larger X'
  - A cable's reactance are small compared to that of overhead lines
- **Increasing the conductor radius decreases X'**
  - Bundling of conductors of HV lines
- **Additional influencing parameters**
  - Line sag, tower geometry, etc.
  - Very elaborate calculations → Tabulated values

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## Example: Line Impedance

- **Calculate the impedance of a two phase T-line built from 1/0 AAC conductor at 60° C. The two phases are 6 ft apart and the line is used on a 60 Hz system.**