From Hadi Saadat, "Power System Analysis", McGraw-Hill, 1999

4.7 INDUCTANCE OF THREE-PHASE TRANSMISSION LINES

4.7.1 SYMMETRICAL SPACING

Consider one meter length of a three-phase line with three conductors, each with radius r, symmetrically spaced in a triangular configuration as shown in Figure 4.7.

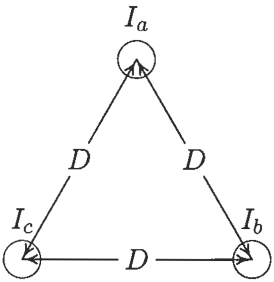

FIGURE 4.7
Three-phase line with symmetrical spacing.

Assuming balanced three-phase currents, we have

$$I_a + I_b + I_c = 0 \tag{4.30}$$

From (4.29) the total flux linkage of phase a conductor is

$$\lambda_a = 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D}\right) \tag{4.31}$$

Substituting for $I_b + I_c = -I_a$

$$\begin{aligned} \lambda_a &= 2 \times 10^{-7}\left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D}\right) \\ &= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \end{aligned} \tag{4.32}$$

Because of symmetry, $\lambda_b = \lambda_c = \lambda_a$, and the three inductances are identical. Therefore, the inductance per phase per kilometer length is

$$L = 0.2 \ln \frac{D}{D_s} \quad \text{mH/km} \tag{4.33}$$

where r' is the geometric mean radius, GMR, and is shown by D_s. For a solid round conductor, $D_s = re^{-\frac{1}{4}}$ for stranded conductor D_s can be evaluated from (4.50). Comparison of (4.33) with (4.23) shows that inductance per phase for a three-phase circuit with equilateral spacing is the same as for one conductor of a single-phase circuit.

4.7.2 ASYMMETRICAL SPACING

Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations. With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced. Consider one meter length of a three-phase line with three conductors, each with radius r. The Conductors are asymmetrically spaced with distances shown in Figure 4.8.

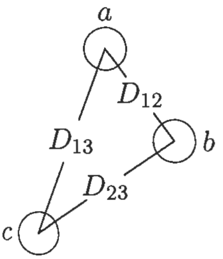

FIGURE 4.8
Three-phase line with asymmetrical spacing.

The application of (4.29) will result in the following flux linkages.

$$
\begin{aligned}
\lambda_a &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{13}} \right) \\
\lambda_b &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{12}} + I_b \ln \frac{1}{r'} + I_c \ln \frac{1}{D_{23}} \right) \\
\lambda_c &= 2 \times 10^{-7} \left(I_a \ln \frac{1}{D_{13}} + I_b \ln \frac{1}{D_{23}} + I_c \ln \frac{1}{r'} \right)
\end{aligned} \tag{4.34}
$$

or in matrix form

$$
\boldsymbol{\lambda} = \boldsymbol{LI} \tag{4.35}
$$

where the symmetrical inductance matrix L is given by

$$
L = 2 \times 10^{-7} \begin{bmatrix} \ln \frac{1}{r'} & \ln \frac{1}{D_{12}} & \ln \frac{1}{D_{13}} \\ \ln \frac{1}{D_{12}} & \ln \frac{1}{r'} & \ln \frac{1}{D_{23}} \\ \ln \frac{1}{D_{13}} & \ln \frac{1}{D_{23}} & \ln \frac{1}{r'} \end{bmatrix} \tag{4.36}
$$

For balanced three-phase currents with I_a as reference, we have

$$
\begin{aligned}
I_b &= I_a \angle 240^\circ = a^2 I_a \\
I_c &= I_a \angle 120^\circ = a I_a
\end{aligned} \tag{4.37}
$$

where the operator $a = 1\angle 120°$ and $a^2 = 1\angle 240°$. Substituting in (4.34) results in

$$\begin{aligned} L_a &= \frac{\lambda_a}{I_a} = 2 \times 10^{-7}\left(\ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{12}} + a\ln\frac{1}{D_{13}}\right) \\ L_b &= \frac{\lambda_b}{I_b} = 2 \times 10^{-7}\left(a\ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2\ln\frac{1}{D_{23}}\right) \\ L_c &= \frac{\lambda_c}{I_c} = 2 \times 10^{-7}\left(a^2\ln\frac{1}{D_{13}} + a\ln\frac{1}{D_{23}} + \ln\frac{1}{r'}\right) \end{aligned} \tag{4.38}$$

Examination of (4.38) shows that the phase inductances are not equal and they contain an imaginary term due to the mutual inductance.

4.7.3 TRANSPOSE LINE

A per-phase model of the transmission line is required in most power system analysis. One way to regain symmetry in good measure and obtain a per-phase model is to consider transposition. This consists of interchanging the phase configuration every one-third the length so that each conductor is moved to occupy the next physical position in a regular sequence. Such a transposition arrangement is shown in Figure 4.9.

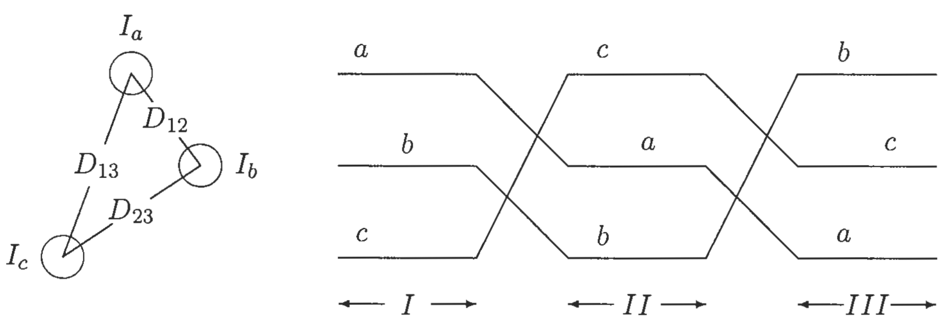

FIGURE 4.9
A transposed three-phase line.

Since in a transposed line each phase takes all three positions, the inductance per phase can be obtained by finding the average value of (4.38).

$$L = \frac{L_a + L_b + L_c}{3} \tag{4.39}$$

Noting $a + a^2 = 1\angle 120° + 1\angle 240° = -1$, the average of (4.38) becomes

$$L = \frac{2 \times 10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{12}} - \ln\frac{1}{D_{23}} - \ln\frac{1}{D_{13}}\right)$$

or

$$L = 2 \times 10^{-7} \left(\ln \frac{1}{r'} - \ln \frac{1}{(D_{12} D_{23} D_{13})^{\frac{1}{3}}} \right)$$

$$= 2 \times 10^{-7} \ln \frac{(D_{12} D_{23} D_{13})^{\frac{1}{3}}}{r'} \qquad (4.40)$$

or the inductance per phase per kilometer length is

$$L = 0.2 \ln \frac{GMD}{D_S} \quad \text{mH/km} \qquad (4.41)$$

where

$$GMD = \sqrt[3]{D_{12} D_{23} D_{13}} \qquad (4.42)$$

This again is of the same form as the expression for the inductance of one phase of a single-phase line. GMD (geometric mean distance) is the equivalent conductor spacing. For the above three-phase line this is the cube root of the product of the three-phase spacings. D_s is the geometric mean radius, GMR. For stranded conductor D_s is obtained from the manufacturer's data. For solid conductor, $D_s = r' = re^{-\frac{1}{4}}$.

In modern transmission lines, transposition is not generally used. However, for the purpose of modeling, it is most practical to treat the circuit as transposed. The error introduced as a result of this assumption is very small.