Thus we can say that zero padding can only make spectrum more dense which does not mean good frequency resolution. More points or samples will produce more frequency components and yet good frequency resolution. This is not to say that good frequency resolution comes from more samples only. You need to consider a good time span over which the signal is well known. If the signal is periodic, you can sample over one period. Taking more samples within this period gives good frequency resolution and yet more frequencies will be detected using the DFT. If the signal is not periodic but its amplitude approaches zero as time reaches a certain limit, then you need to sample the signal for the period of time up to that limit. This is what is known as the record length in digital signal processing.

7.10 End of Chapter Problems

EOCP 7.1

A continuous time signal has f_m as its highest frequency. If $f_m = 1$ kHz and we desire sampling the signal at 10 times f_m, what would be the record length and the number of samples if the frequency resolution is to be 10 Hz?

EOCP 7.2

Find the circular convolution between the signals

1. $u(n)$ for $0 \le n \le 5$ and $e^{-n/.1}$ for $0 \le n \le 5$
2. $5\sin(n\pi/3)$ for $0 \le n \le 4$ and e^{-n} for $0 \le n \le 4$
3. $e^{-n/2}$ and itself for $0 \le n \le 10$
4. $x(n) = 1$ and itself for $0 \le n \le 5$
5. $\cos(n\pi/6)$ for $0 \le n \le 6$ and $\delta(n)$ for $0 \le n \le 6$

EOCP 7.3

1. Find the discrete Fourier transform $X(e^{j\theta})$ of $x(n) = e^{-n/3}$ for $0 \le n \le 7$.
2. Sample $X(e^{j\theta})$ at $\theta = 2\pi k/N$ with $N = 16$.
3. Find the first 4 values $X(0)$,…, $X(3)$ using the DFT equation.
4. Find all eight values for $X(k)$ and compare with the values found in part 2.
5. If $x(n)$ in part 1 is an input to the linear system given by $h(n) = e^{-n/6}$ for $0 \le n \le 3$, use convolution to find $y(n)$, the output of the system.
6. Use the DFT to find $y(n)$ in part 5.

EOCP 7.4

Consider the signal

$$x(n) = \begin{cases} \cos(n\pi/6) & 0 \le n \le 5 \\ 0 & \text{otherwise} \end{cases}$$

1. Let N = 8, 16, 32 and 64. Find the spectrum for $x(n)$ using the DFT.
2. What conclusion can you draw by completing part 1?
3. If $x(n) = \cos(n\pi/6)$ for $0 \le n \le 63$, plot the spectrum for $x(n)$ in this case.
4. Compare the results of part 1 and part 3.

EOCP 7.5

Find the DFT of the following signals where n is taken in the interval $0 \le n \le N - 1$. A is a constant.

1. $A\delta(n)$
2. A
3. $A\sin(2\pi n/N)$
4. $A\cos(2\pi n/N)$

EOCP 7.6

Consider the signal

$$x(t) = \sin(600\pi t) + \sin(1000\pi t)$$

1. What is the period of $x(t)$?
2. What is the minimum sampling frequency?
3. Sample $x(t)$ for one period at $f_s = 10 f_m$ and plot $x(n)$.
4. Find the DFT of $x(n)$ in 3.
5. Repeat 3 over two periods. What do you observe? Keep N as in 3.
6. Find the DFT of $x(n)$ in 5. What do you notice?

EOCP 7.7

Use the DFT to find the energy spectrum density for the signals

1. $x(n) = \sin\left(\frac{2\pi n}{11}\right) \quad 0 \le n \le 15$
2. $x(n) = e^{\frac{-n}{3}} \sin\left(\frac{2\pi n}{11}\right) 0 \le n \le 15$

EOCP 7.8

Use the DFT to find the cross-correlation between the signals given in EOCP 7.7.

EOCP 7.9

Use the DFT to approximate the Fourier transform for the signals

1. $x(t) = e^{-t}u(t)$
2. $x(t) = 20\cos\left(\frac{2\pi t}{13}\right)$
3. $x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

EOCP 7.10

Consider the system

$$h(n) = e^{-n} \qquad 0 \le n \le 10$$

1. If $x(n) = u(n)$ for $0 \le n \le 5$, find $y(n)$ using linear convolution via the DFT.
2. Find the impulse response for the system.

EOCP 7.11

Consider the system

$$h(t) = e^{\frac{-t}{3}}u(t)$$

1. If the input is $x(t) = e^{-5t}u(t)$, use the DFT to find $y(n)$.
2. If the input is $x(t) = 10sin(2\pi(500)t) + 10$, find the output $y(n)$ using the DFT.
3. Is there any dc component in the output? Use the DFT to check.

EOCP 7.12

Consider the signals

$$x(t) = \sin(2\pi t) + \cos\left(\left(3\pi/4\right)t\right)$$

$$x(t) = e^{-10t}\sin(2\pi t) + \cos\left(3/4t\right)$$

1. Are the signals periodic?
2. Find the Fourier series coefficient and/or the Fourier transform (approximation) using the DFT for the signals.
3. What is the average power/total energy in the signals $x(t)$?

EOCP 7.13

Consider the signal

$$x(t) = e^{\frac{-t}{10}}u(t)$$

1. Find the total energy in the signal using the DFT.
2. If $x(t)$ is multiplied by $\sin(t)$, what would be the approximation to the total energy in the signal using the DFT.

EOCP 7.14

Consider the signal

$$x(t) = e^{-10t}\sin(t)u(t)$$

as an input to the system

$$h(t) = e^{-4t}u(t)$$

1. Find $y(n)$ using convolution and the DFT.
2. Subdivide the input signal into blocks and use block filtering to find $y(n)$ again.
3. Compare the results in 1 and 2.
4. Find the total energy in both signals using the DFT.

EOCP 7.15

Consider the signal

$$x(t) = \cos(2\pi(300)t)$$

1. Find the approximation to the Fourier transform of $x(t)$.
2. Use the Hamming windowing method and repeat part 1.
3. Use the Hamming windowing method and repeat part 1.
4. Comment on the results.