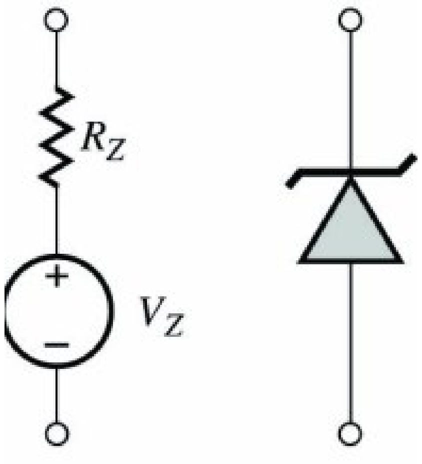
Week 3: Diode Application Circuits

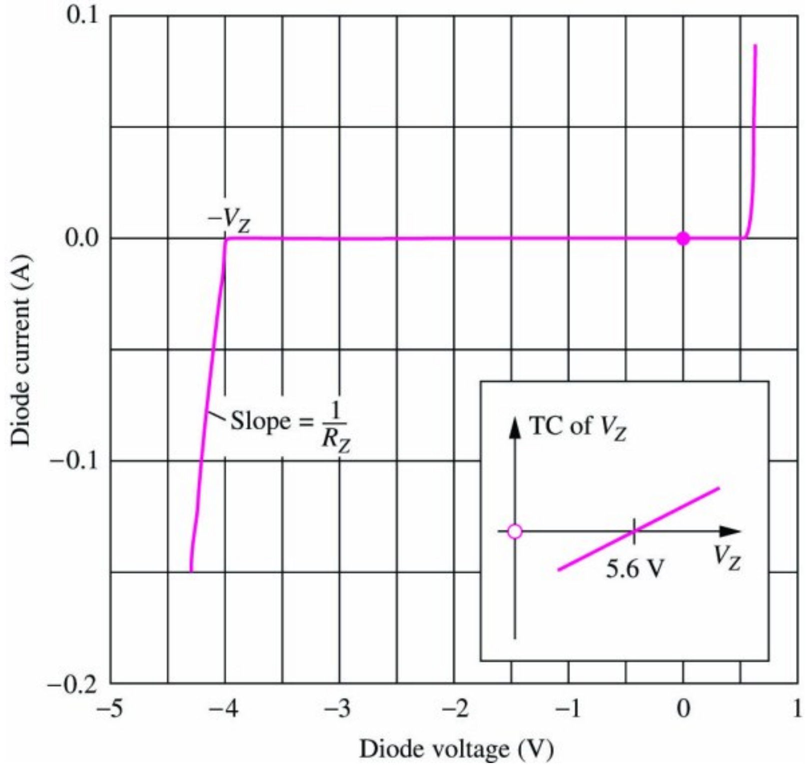
Topics to cover …

- Voltage Regulation - Zener Diode
- Rectifiers
- DC-to-DC converters
- Wave shaping circuits
- Photodiode and LED

- Reading Assignment:
  Chap 3.13-3.16, 3.18 of Jaeger & Blalock

Breakdown Region Diode Model

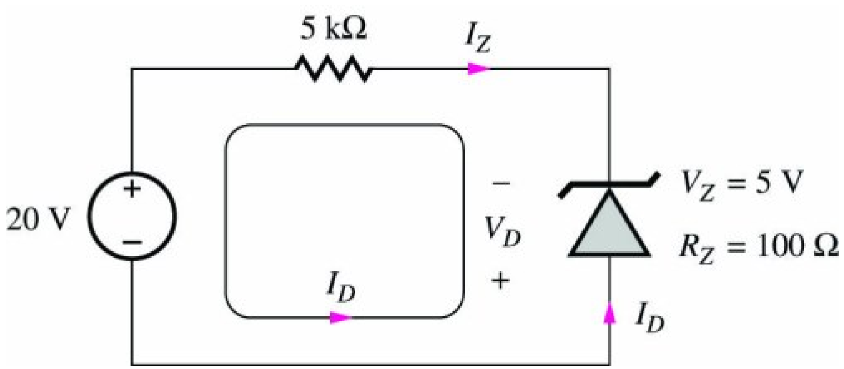


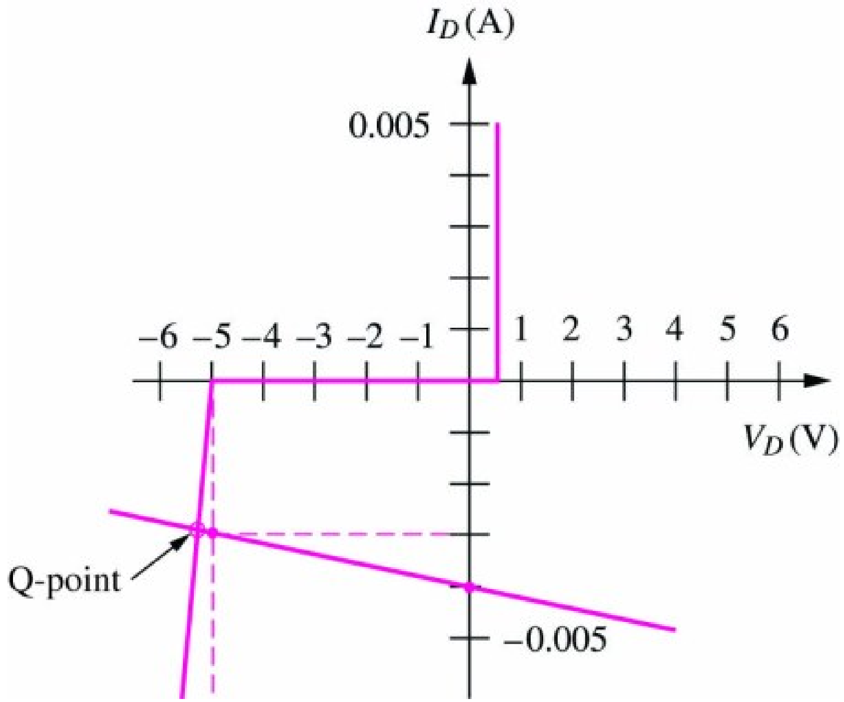


In breakdown, the diode is modeled with a DC voltage source, V_Z, and a series resistance, R_Z. R_Z models the slope of the *i-v* characteristic.

Diodes designed to operate in reverse breakdown are called **Zener diodes** and use the indicated symbol

Example 1 – Graphic Analysis





Load-line equation :
(KVL along the loop)

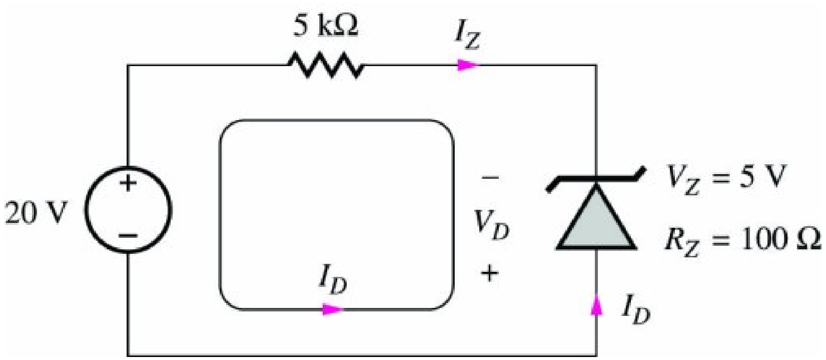
$$V_D + 5000 I_D + 20 = 0$$

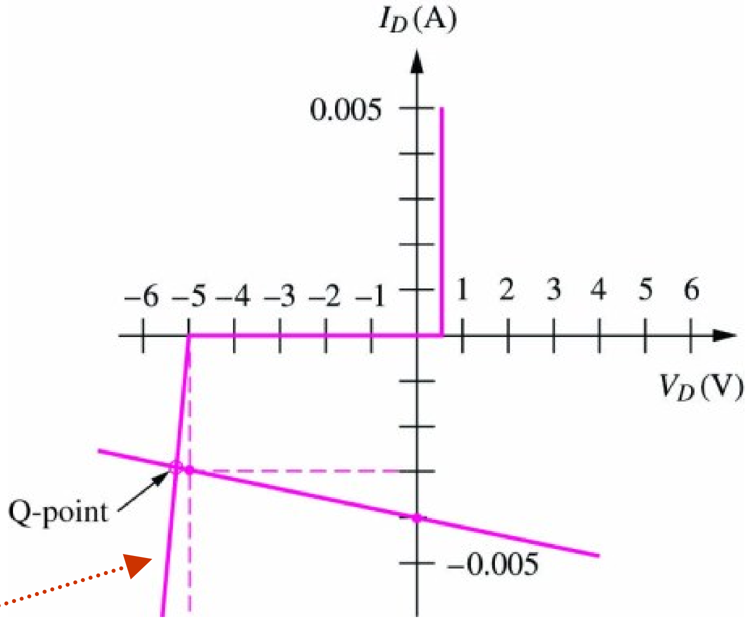
Using graphic analysis:

Choose 2 points (0V, -4mA) and (-5V, -3mA) to draw the load line.

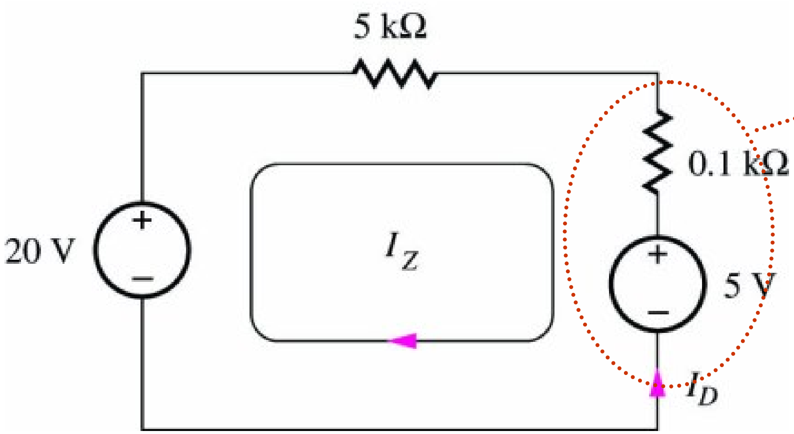
It intersects with *i-v* characteristic at **Q-point (-2.9 mA, -5.2 V)**.

Example 2 – Piecewise Linear Model





Using piecewise linear model:



$$I_Z = -I_D > 0$$

Assume in breakdown region

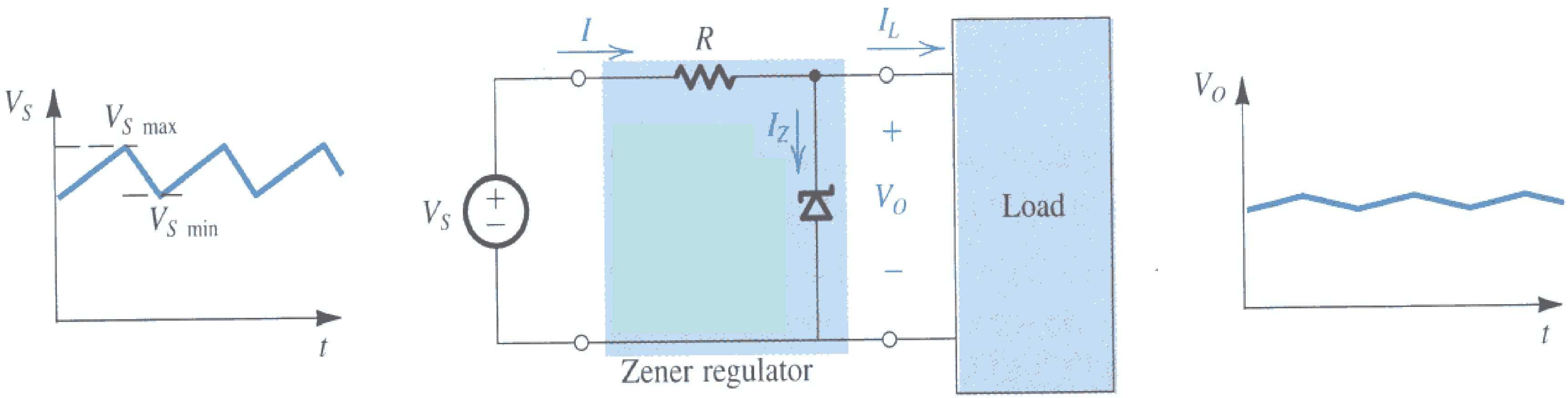
$$-20 + 5100 I_Z + 5 = 0$$

$$I_Z = \frac{(20-5)\text{V}}{5100\Omega} = 2.94\text{mA}$$

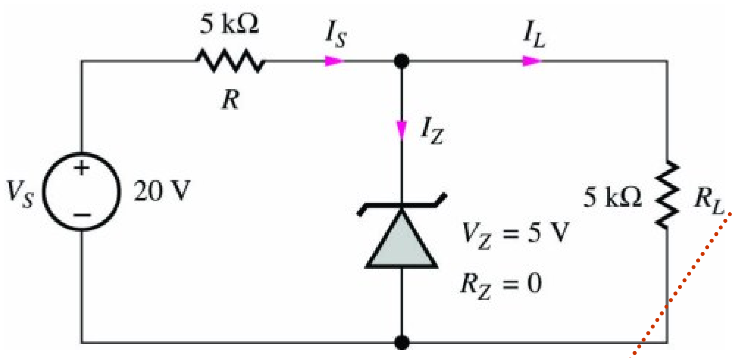
Since I_Z >0 (I_D <0), solution is consistent with Zener breakdown assumption.

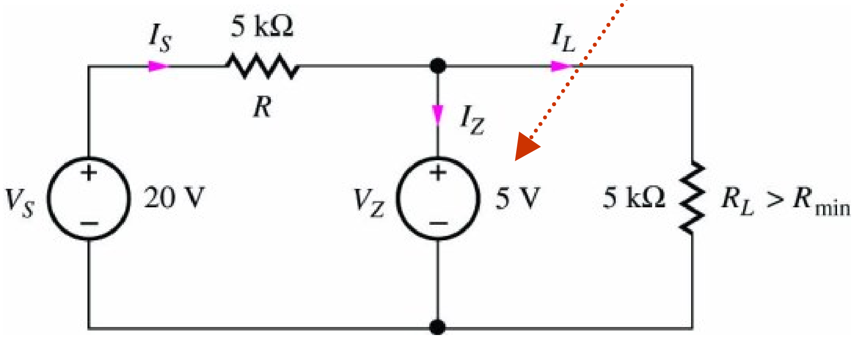
Voltage Regulator

- Function: to maintain a constant voltage across the load when the source voltage or the load current vary.



Voltage Regulator





Zener diode keeps voltage across load resistor constant

Use constant voltage drop model:

$$I_S = \frac{V_S - V_Z}{R} = \frac{(20-5)\text{V}}{5\text{k}\Omega} = 3\text{mA}$$

$$I_L = \frac{V_Z}{R_L} = \frac{5\text{V}}{5\text{k}\Omega} = 1\text{mA}$$

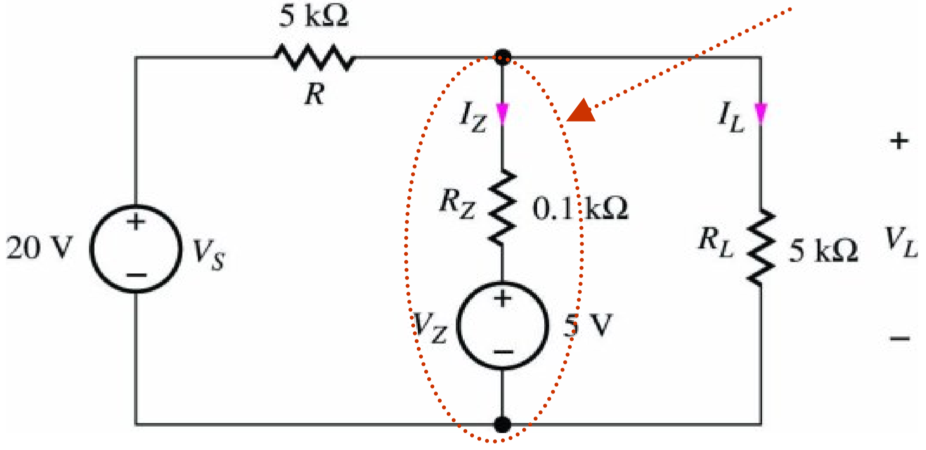
$$I_Z = I_S - I_L = 2\text{mA}$$

For proper regulation, $I_Z > 0$. If $I_Z < 0$, Zener diode no longer controls voltage across load resistor and the regulator is said to have "dropped out of regulation".

$$\therefore I_Z = \frac{V_S}{R} - V_Z\left(\frac{1}{R} + \frac{1}{R_L}\right) > 0 \qquad R_L > \frac{R}{\left(\frac{V_S}{V_Z} - 1\right)} = R_{min}$$

Example 3

Zener diode model



Problem: Find V_L and I_Z using the piece-wise linear model.

Given data:

V_S=20 V, R=5 kΩ,

R_Z= 0.1 kΩ, V_Z=5 V

Analysis:

KCL at output node:

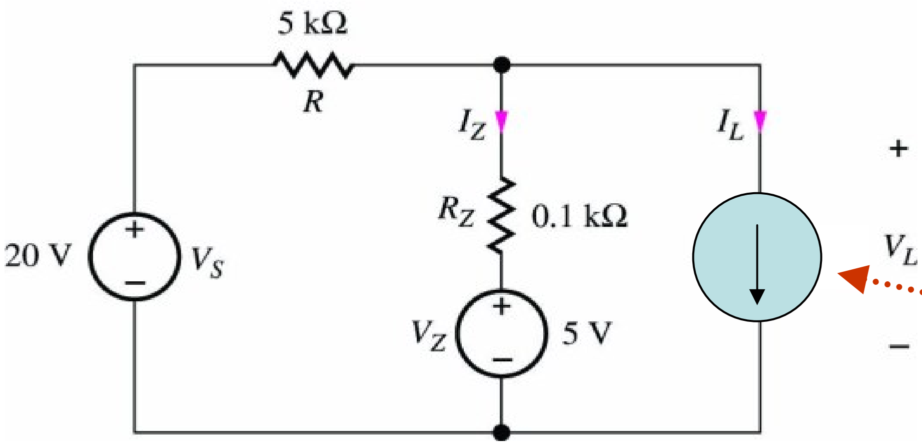
$$\frac{V_L - 20}{5000} + \frac{V_L - 5}{100} + \frac{V_L}{5000} = 0$$

$$\therefore V_L = 5.19\text{V}$$

$$I_Z = \frac{V_L - 5}{100} = \frac{5.19 - 5}{100} = 1.9\text{mA} > 0$$

Diode is actually operating in breakdown region as assumed.

Line and Load Regulation



Load is modeled as a current drain

By superposition: $$V_L = \frac{R}{R+R_z}V_Z + \frac{R_z}{R+R_z}V_S - (R_z // R)I_L$$

$$Line\ \ regulation \equiv \frac{\partial V_L}{\partial V_S} = \frac{R_z}{R+R_z}$$

(characterizes how sensitive output voltage is to input voltage changes)

$$Load\ \ regulation \equiv \frac{\partial V_L}{\partial I_L} = -(R_z // R)\,[\Omega]$$

(characterizes how sensitive output voltage is to load current changes)

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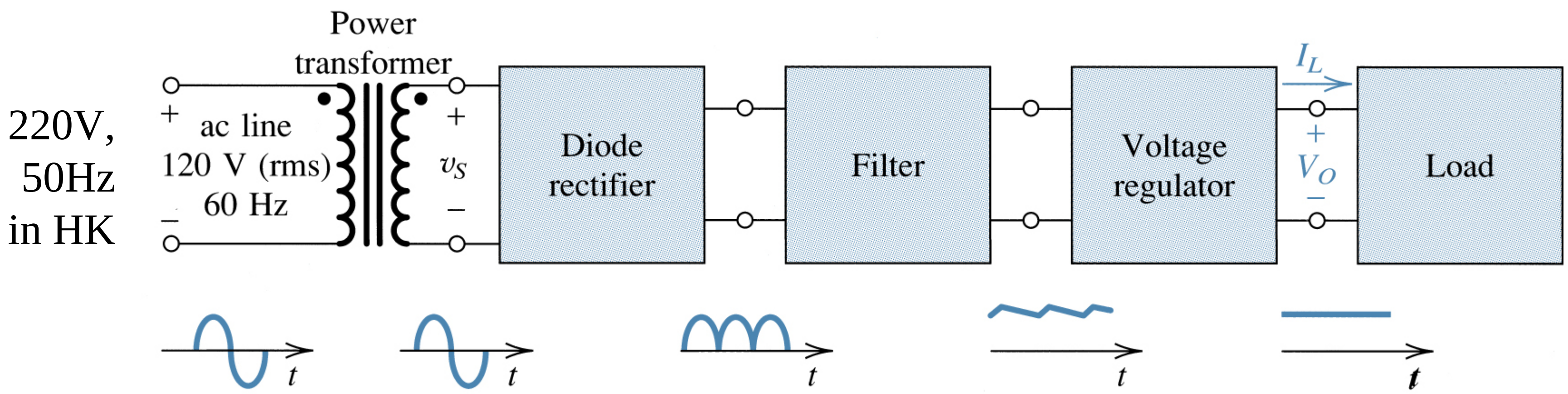
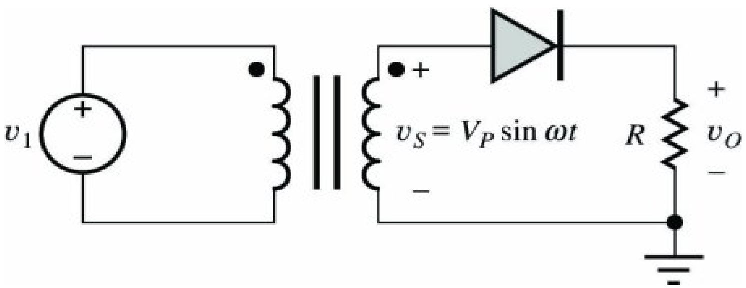
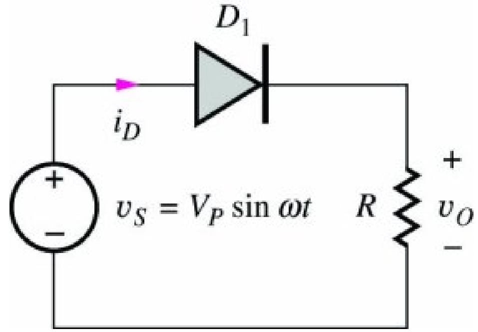
AC-DC Converter



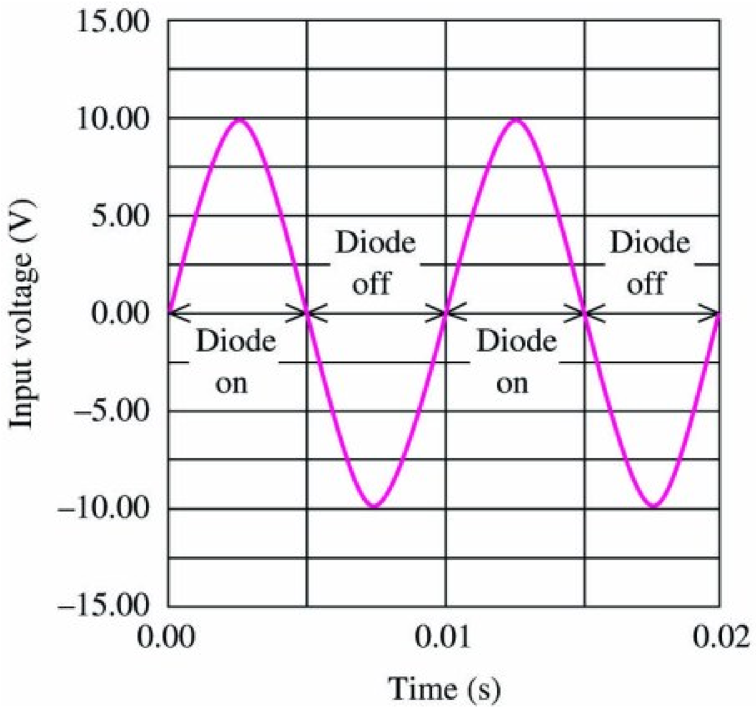
- Transformer:
  - step down AC voltage amplitude
  - Isolate equipment from power-line
- Rectifier:
  - converts an ac input to a unipolar output
- Filter:
  - convert the pulsating input to a nearly constant dc output
- Regulator:
  - Reduce the ***ripple*** of the dc voltage

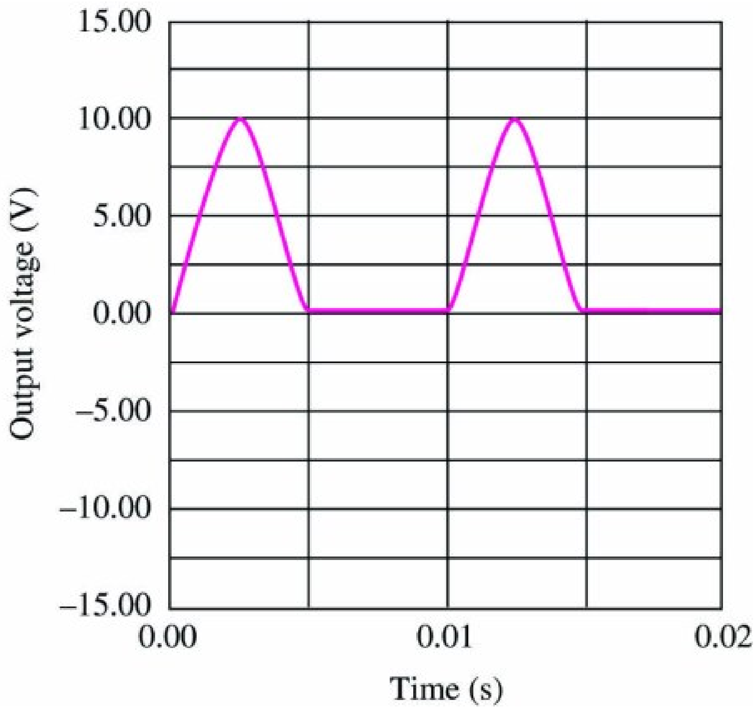
Half-Wave Rectifier





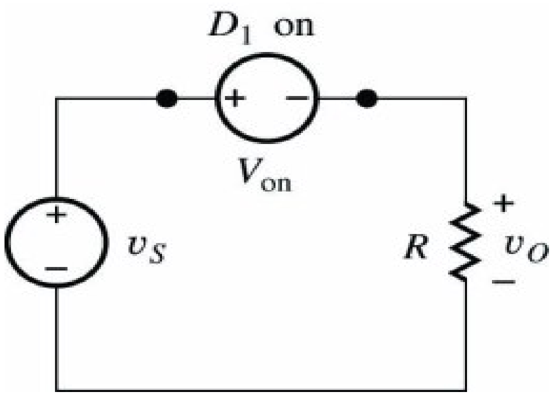
Using ideal Diode model:





Half-Wave Rectifier

Using Constant Voltage Drop model:



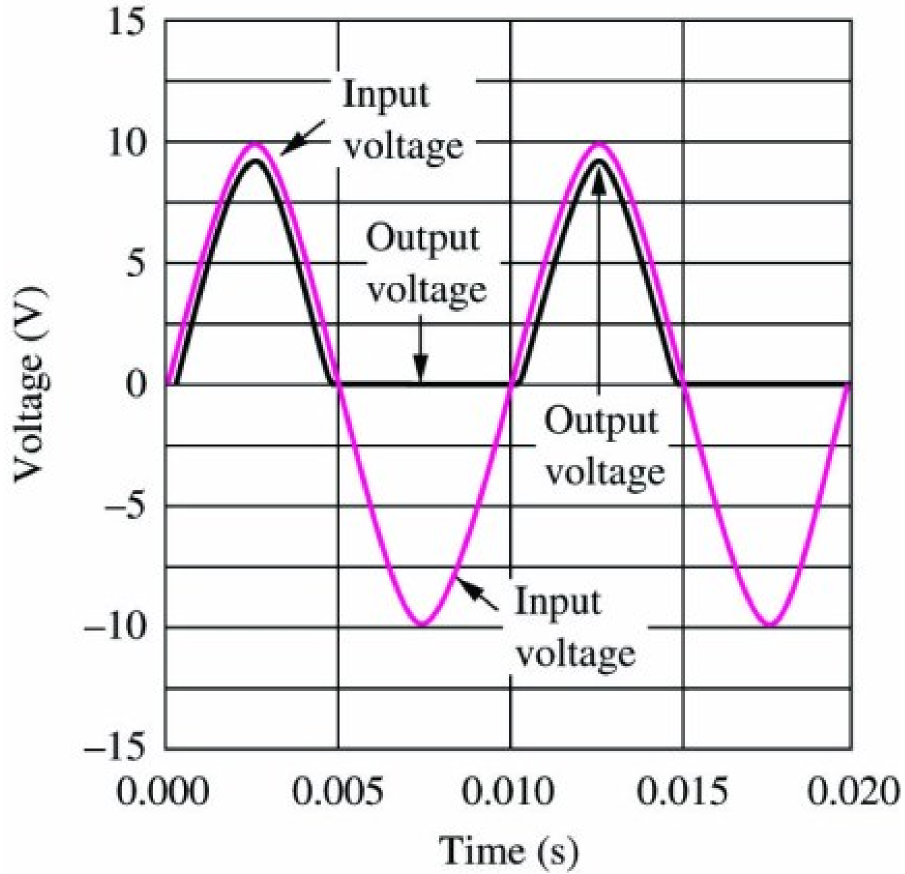
When diode is ON:

$$v_o = (V_P \sin \omega t) - V_{on}.$$

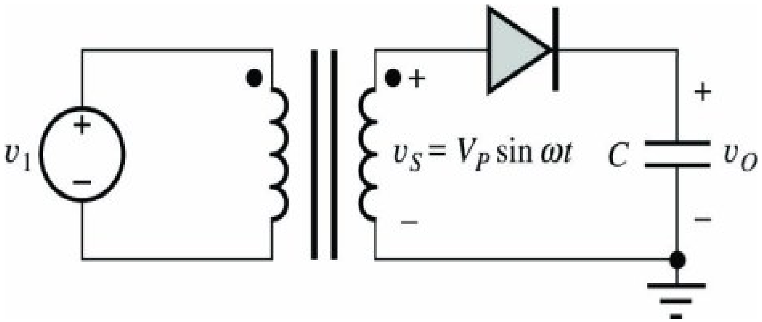
When diode is OFF:

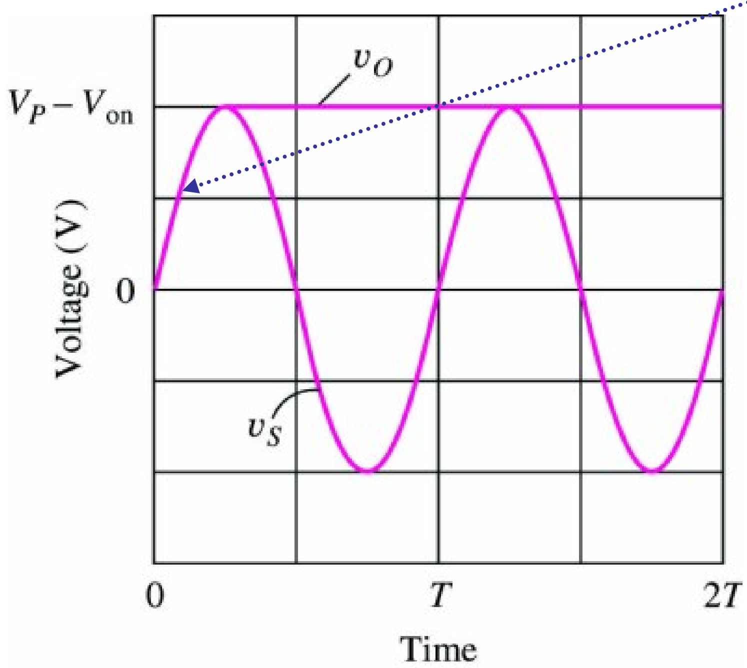
$$v_o = 0.$$

Time-varying components in circuit output can be suppressed using a filter capacitor.



Peak Detector Circuit





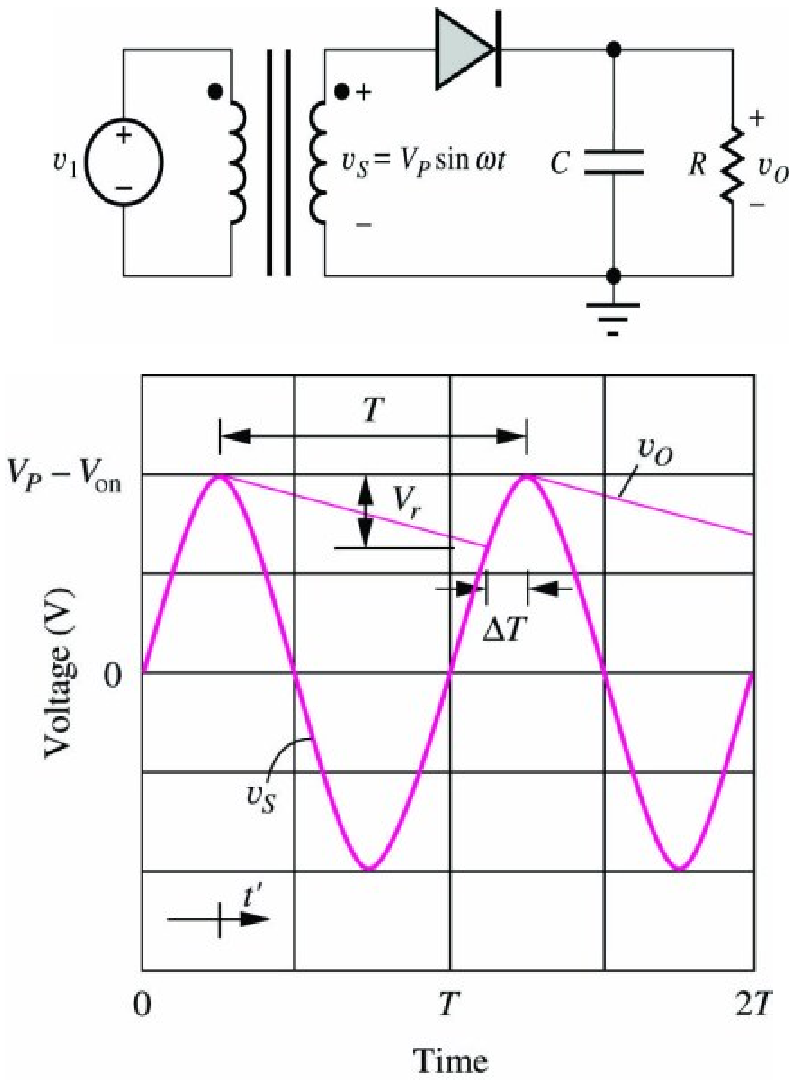
Replace R with C

As input voltage rises, diode is on and capacitor (initially discharged) charges up to V_s -V_{on}

At the peak of input, diode current tries to reverse, diode cuts off, capacitor has no discharge path and retains constant, thereby providing a dc output voltage:

$$V_{dc} = V_P - V_{on}$$

Half-Wave Rectifier with *RC* Load

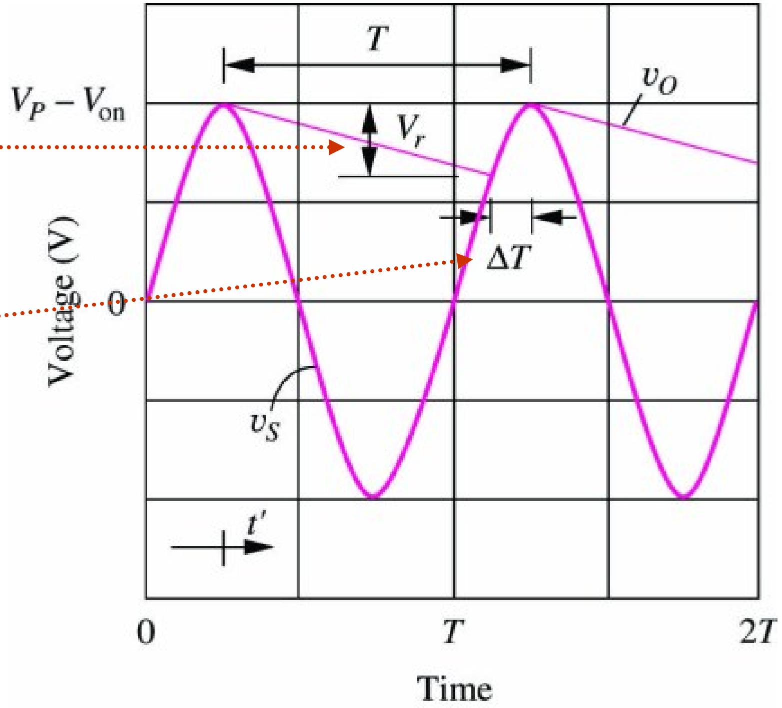


- As input voltage rises during first quarter cycle,
  - diode is on
  - capacitor charges up to peak value of input voltage
- After the peak of input,
  - Input voltage < V_P
  - Diode cuts off
  - Capacitor discharges exponentially through R
  - Discharge continues till input voltage exceeds output voltage which occurs near peak of next cycle.
- This process then repeats once every cycle

Half-Wave Rectifier with *RC* Load

Output voltage is not constant as in ideal peak detector, but has ***ripple voltage*** V_r.

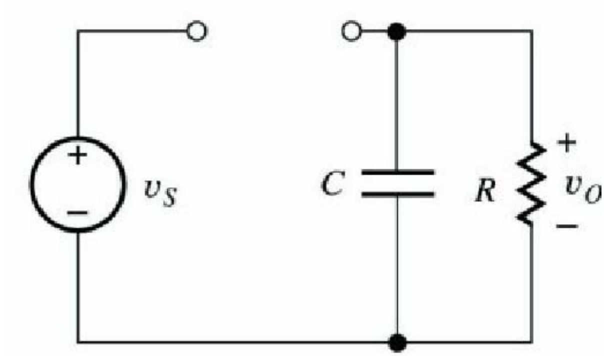
Diode conducts for a short time D*T* called ***conduction interval*** during each cycle and its angular equivalent is called ***conduction angle*** θ_c.



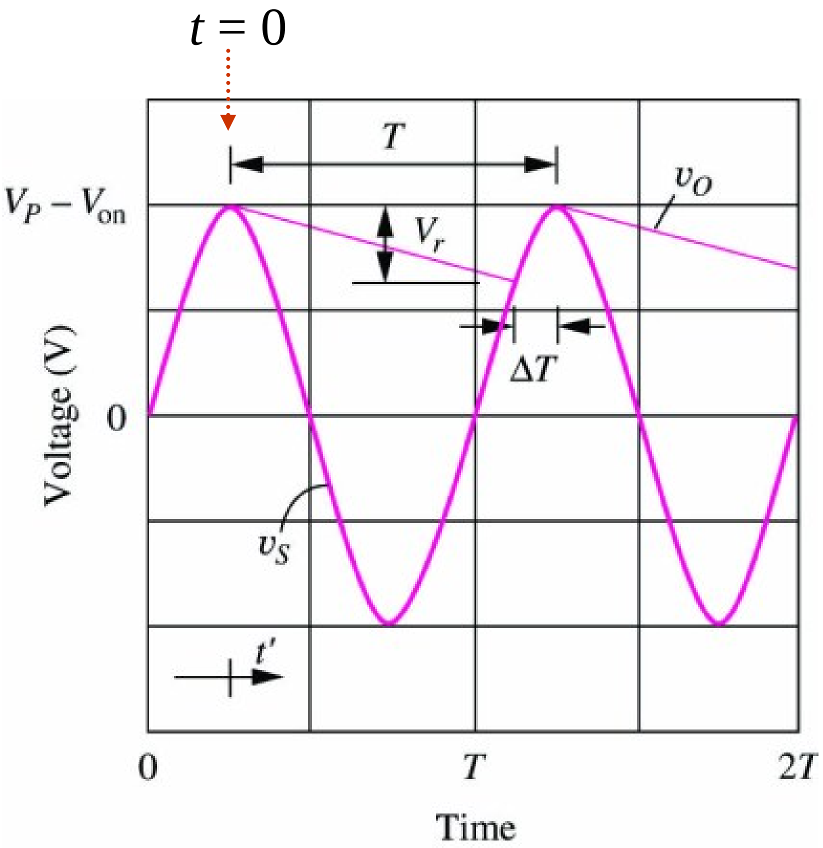
During discharge period,

$$v_o(t) = (V_P - V_{on})\exp(-t/RC)$$
$$\cong (V_P - V_{on})(1 - t/RC) \quad \text{for } t << RC$$

$$V_r \cong (V_P - V_{on})\frac{T - \Delta T}{RC} \cong (V_P - V_{on})\frac{T}{RC} \quad \text{for } \Delta T << T$$



Half-Wave Rectifier with *RC* Load



To find the conduction period,

$$(V_P - V_{on})\exp(-\frac{T-\Delta T}{RC}) = V_P \cos\omega(T-\Delta T) - V_{on}$$

Assume $\Delta T << T$,
It can be solved that,

$$\Delta T \cong \frac{1}{\omega}\sqrt{\frac{2T}{RC}\frac{(V_P - V_{on})}{V_P}} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}}$$

$$\theta_C = \omega\Delta T = \sqrt{\frac{2V_r}{V_P}}$$ where $$\omega = \frac{2\pi}{T}$$

Example 4

Problem: Find dc output voltage, output current, ripple voltage, conduction interval, conduction angle of RC loaded half-wave rectifier.

Given data: secondary voltage V_{rms} =12.6 (60 Hz), R = 15 Ω, C=25,000μF, V_{on} = 1 V

Analysis:

$$V_{dc} = V_P - V_{on} = (12.6\sqrt{2} - 1)\text{V} = 16.8\text{V}$$

$$I_{dc} = \frac{V_P - V_{on}}{R} = \frac{16.8\text{V}}{15\Omega} = 1.12\text{A}$$

Using discharge interval T=1/60 s,

$$V_r \cong (V_P - V_{on})\frac{T}{RC} = 0.747\text{V}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_P}} = 0.290\text{ rad or } 16.6°$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{\theta_c}{2\pi f} = \frac{0.29}{120\pi} = 0.769\text{ms}$$

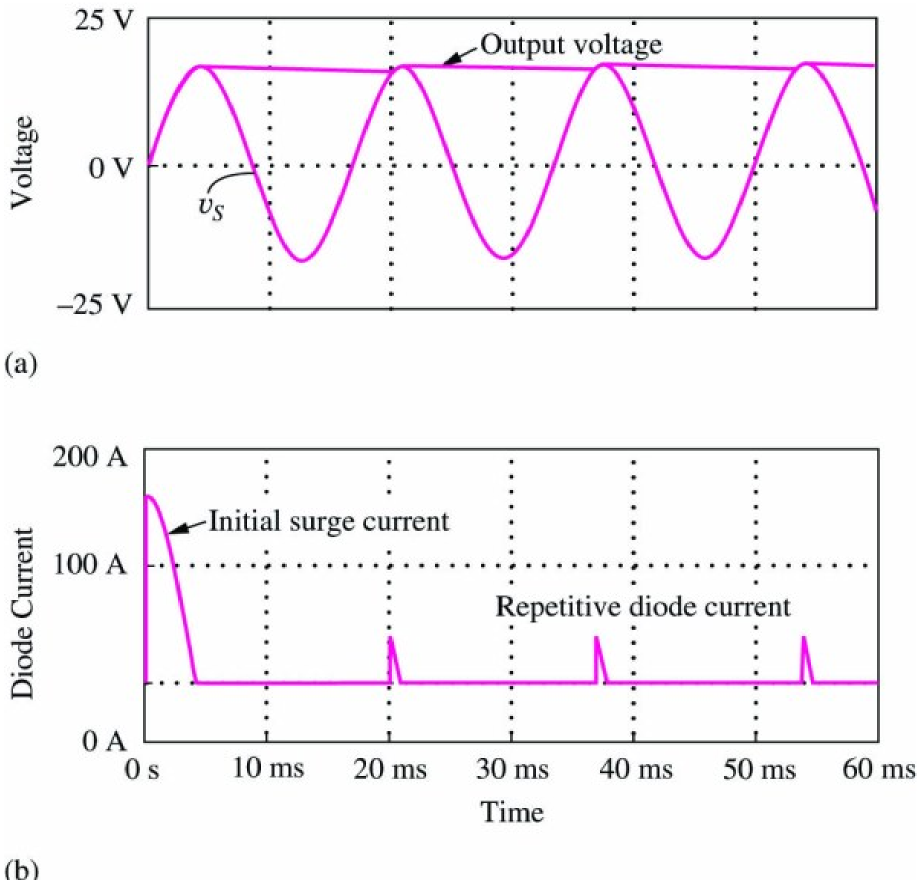
Check of result:

The hidden assumptions are justified:

$$\Delta T << T = 1/60 = 16.7ms$$

$$T << RC = 375ms$$

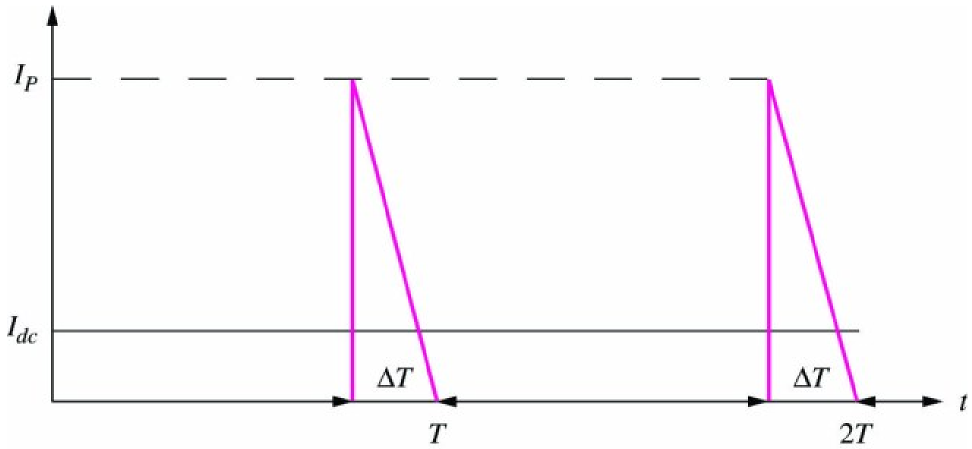
Peak Diode Current



- Nonzero current exists in diode for only a very small fraction of period T
- An almost constant dc current flows out of filter capacitor to load
- Total charge lost from capacitor in each cycle is re-supplied by diode during the conduction interval

Peak Diode Current

- Let's model the repetitive current pulse is modeled as triangle of height I_P and width ΔT,

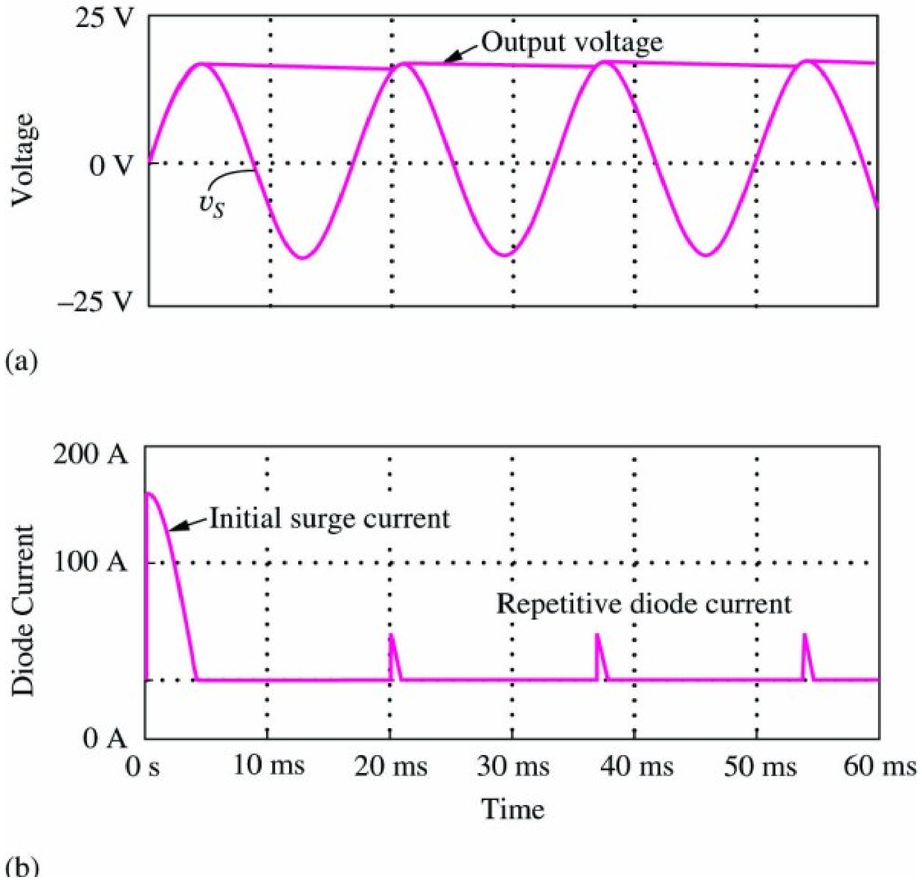


Charge supplied during diode conduction: $$Q = I_P \frac{\Delta T}{2}$$

Charge transferred to the load in each cycle:

$$Q \approx I_{dc} T \qquad \therefore I_P = I_{dc} \frac{2T}{\Delta T} \qquad \left(i \equiv \frac{dQ}{dt} \Rightarrow Q = \int i(t)dt \right)$$

Surge Current



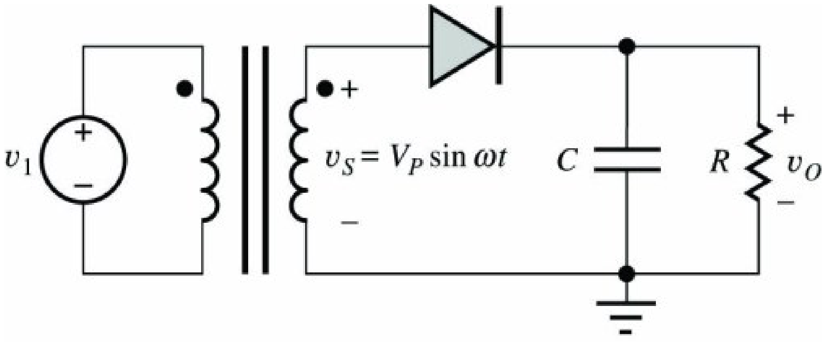
During first quarter cycle,

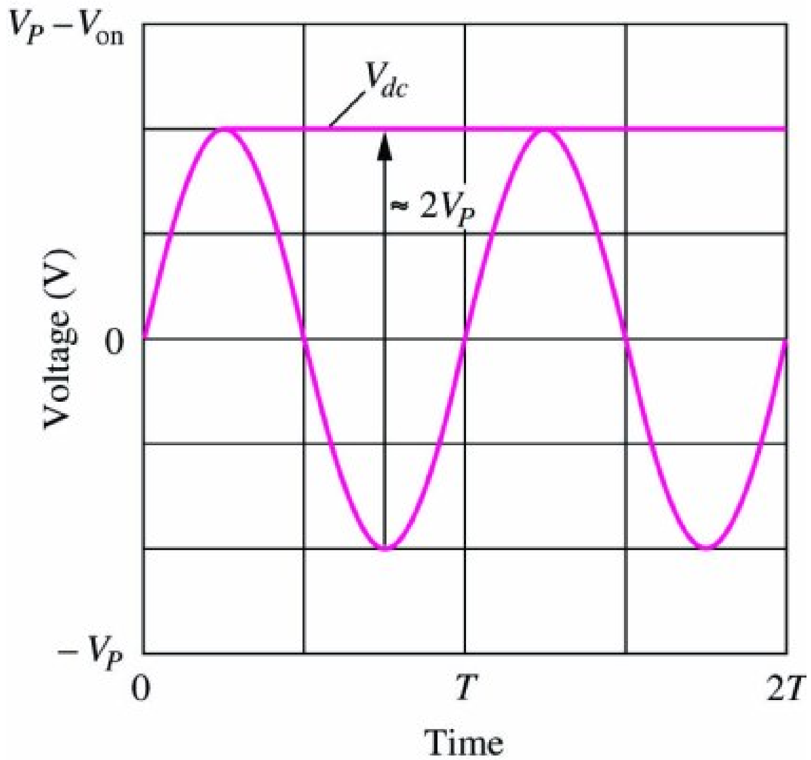
$$i_d(t) = i_c(t) \cong C\left(\frac{d}{dt} V_P \sin \omega t\right)$$
$$= \omega C V_P \cos \omega t$$

Peak values of this initial surge current occurs at $t = 0^+$:

$$I_{SC} = \omega C V_P$$

Peak Inverse Voltage (PIV)



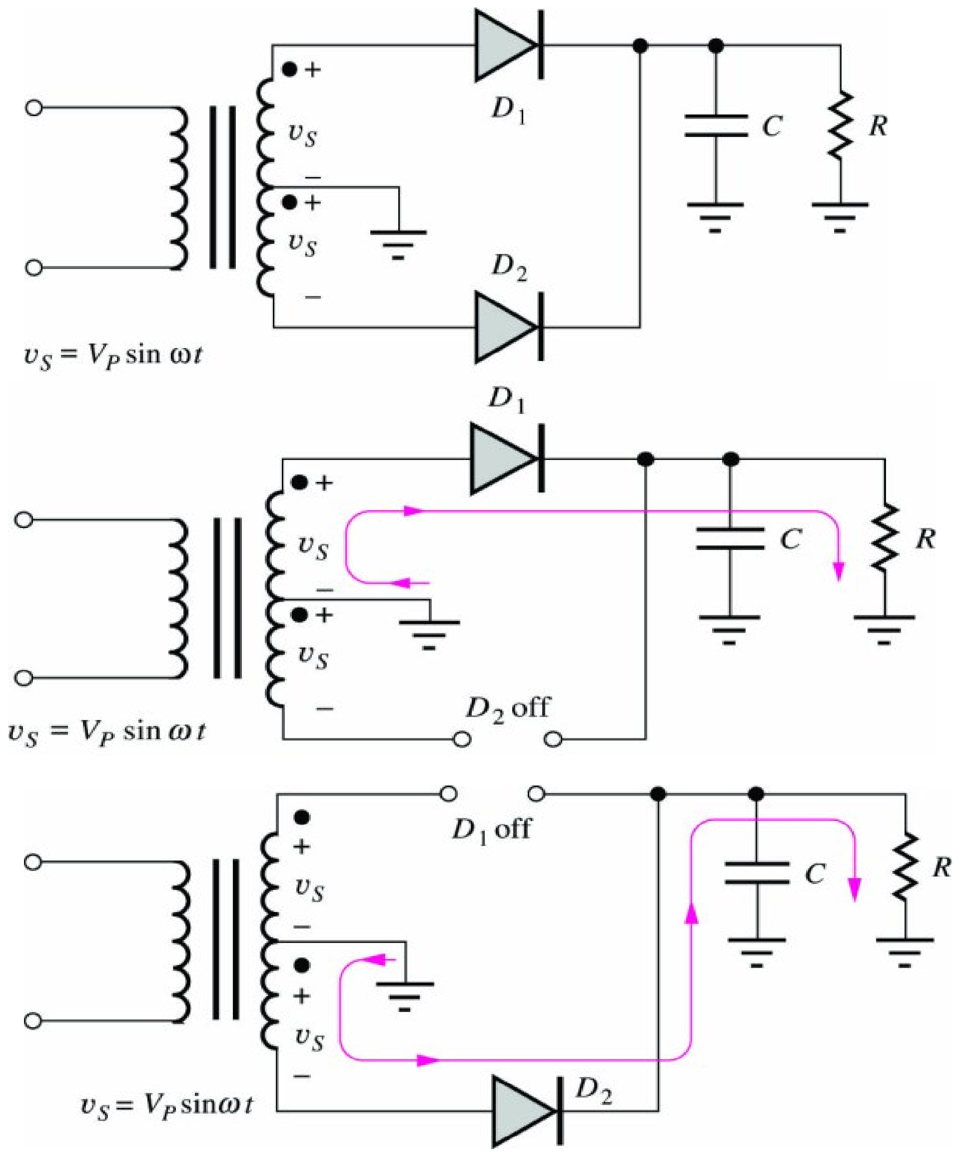


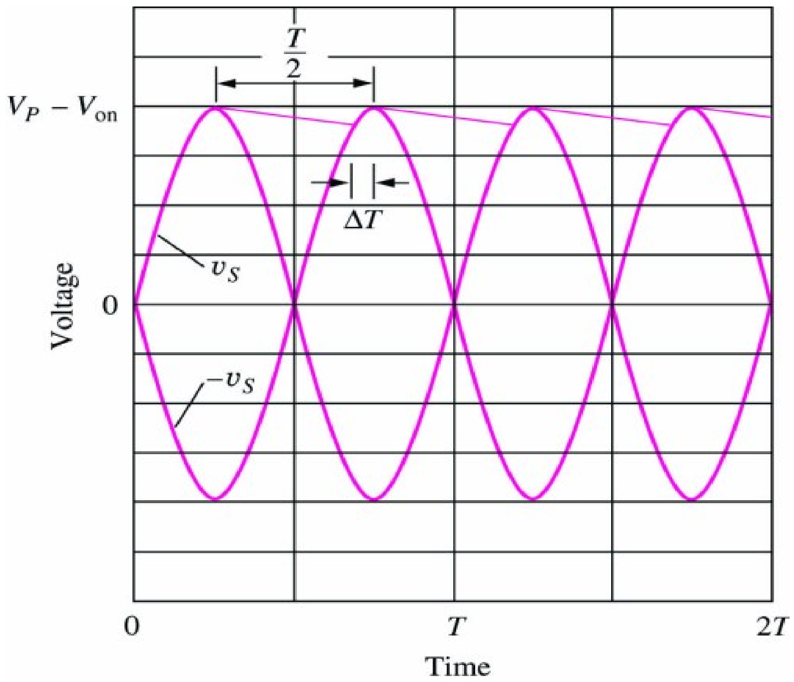
When diode is off, reverse-bias across diode is V_{dc} - v_s. When v_s reaches negative peak, the diode sees the ***peak inverse voltage*** (**PIV**)

$$\text{PIV} \geq V_{dc} - v_S^{\min} = V_P - V_{on} - (-V_P) \cong 2V_P$$

The diode should have a breakdown voltage greater than the PIV in order to avoid entering breakdown region.

Full-Wave Rectifier





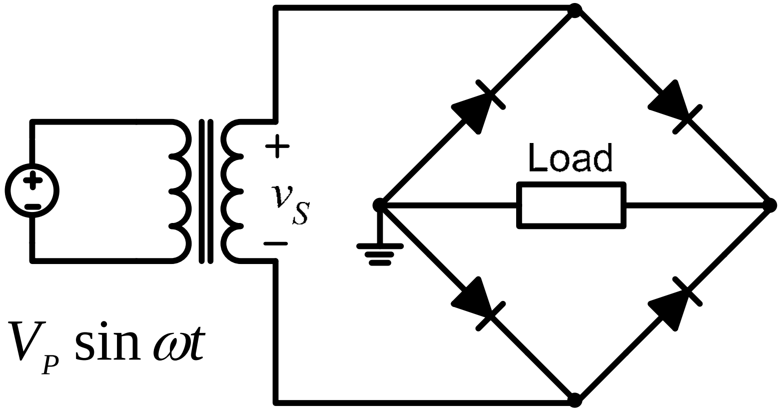
- Capacitor discharge time halved → Require half the filter capacitance to achieve a given ripple voltage
- Output DC = V_P - Von
- PIV= $2V_P$-Von ≈ $2V_P$

Capacitor Size

(b)

A 0.175F capacitor

Full-Wave Bridge Rectifier

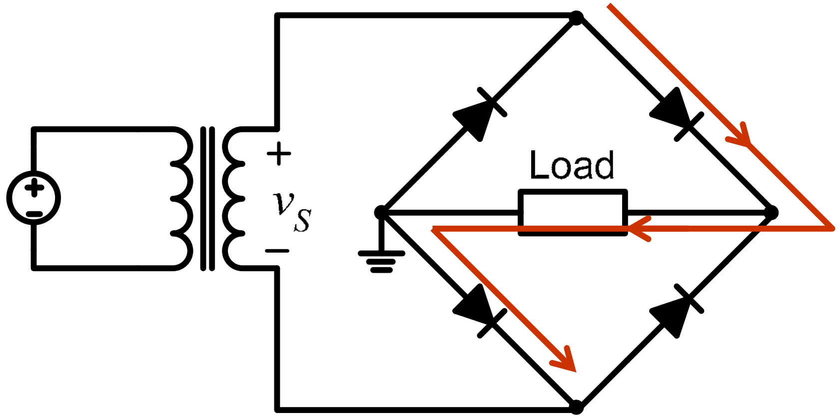


$$v_S = V_P \sin \omega t$$

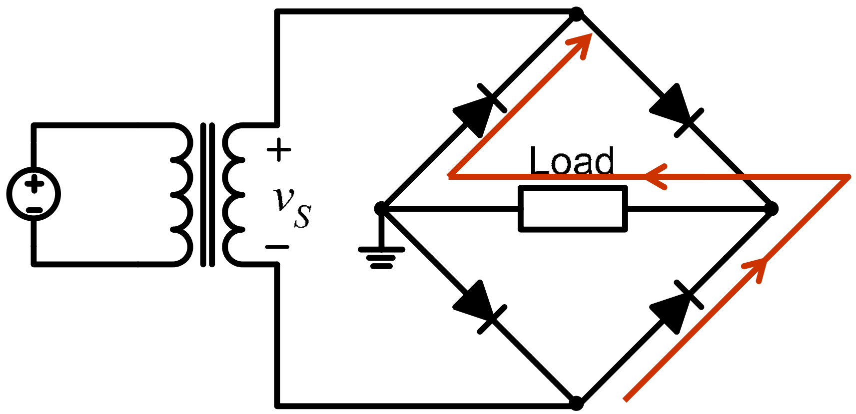
- Current always flows in one direction through the load regardless of whether v_S is positive or negative.
- Which direction?

Full-Wave Bridge Rectifier

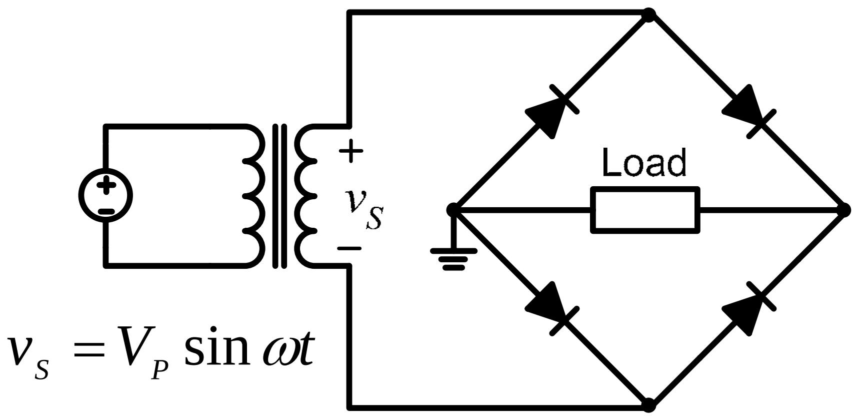
- v_S positive:



- v_S negative:



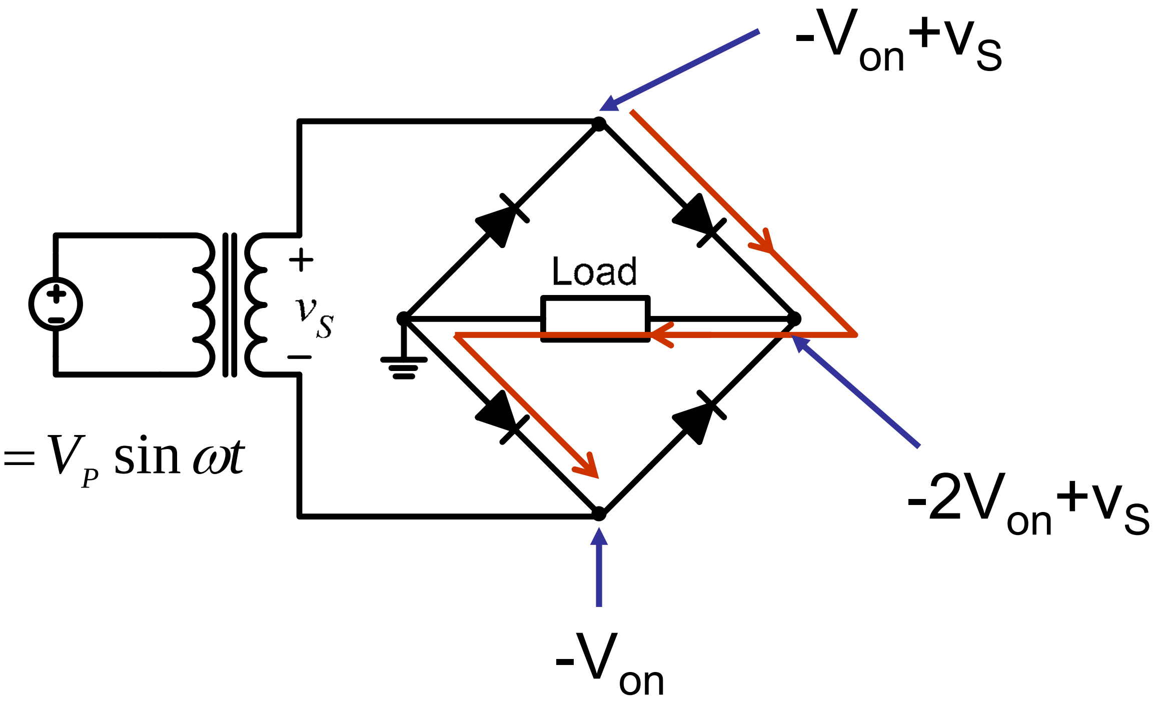
Full-Wave Bridge Rectifier



- Full wave rectification is achieved without center-tapped transformer by using two extra diodes
- Filter capacitance is halved compared to that of the half-wave rectifier
- Two diodes in either conduction path
- Output DC = V_P - $2V_{on}$

Reduced Peak Inverse Voltage

(v$_S$ positive)



$v_S = V_P \sin \omega t$

- PIV = Max($-V_{on}+v_S$) = $V_p - V_{on} \approx V_P$
- So diodes with lower breakdown voltage can be used → Cheaper

Rectifier Topology Comparison

	Output DC Voltage	**PIV**	**Filter capacitance for a same ripple**
Half-Wave	$V_P - V_{ON}$	$2V_P - V_{ON}$	$C = \frac{(V_P - V_{on})}{V_r}\frac{T}{R}$
Center-Tapped Full-Wave	$V_P - V_{ON}$	$2V_P - V_{ON}$	C/2
Bridge Rectifier Full-Wave	$V_P - 2V_{ON}$	$V_P - V_{ON}$	C/2

- Filter capacitor is a major factor in determining cost, size and weight in design of rectifiers.
- Low PIV is important in high-voltage circuits.

Example 5

Problem: Design rectifier with given specifications.
Specifications: V_{dc} =15 V, V_r < 0.15 V, I_{dc} = 2 A,
Assumptions: Von=1V, input freq=50Hz
Analysis: Use full-wave bridge rectifier

$$V_P = V_{dc} + 2V_{on} = 15 + 2 = 17\text{V}$$ (V_P of the transformer output)

$$C = \frac{(V_P - V_{on})}{V_r}\frac{T}{2R} = I_{dc}\left(\frac{T/2}{V_r}\right) = 2\text{A}\left(\frac{1}{100}\text{s}\right)\left(\frac{1}{0.15\text{V}}\right) = 0.133\text{F}$$

$$\Delta\text{T} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{50\cdot 2\pi}\sqrt{\frac{2(0.15\text{V})}{17\text{V}}} = 0.423\text{ms}$$

$$I_P = I_{dc}\left(\frac{2}{\Delta\text{T}}\right)\left(\frac{T}{2}\right) = 2\text{A}\frac{(1/50)\text{s}}{0.423\text{ms}} = 94.6\text{A}$$

$$\text{PIV} = V_P - V_{ON} = 16\text{V}$$

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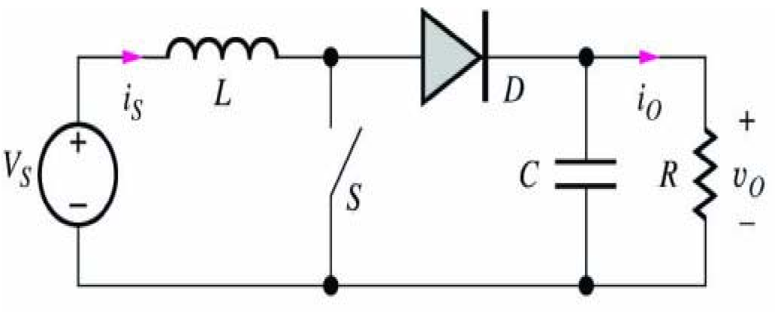
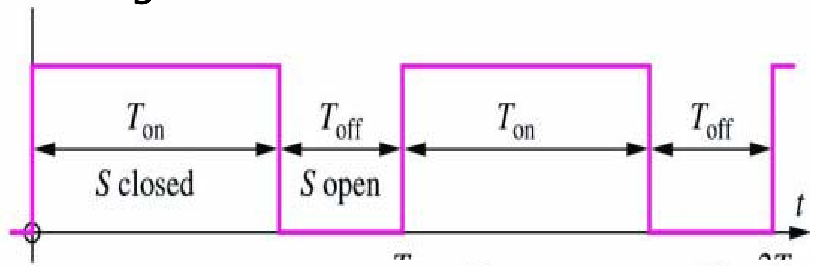
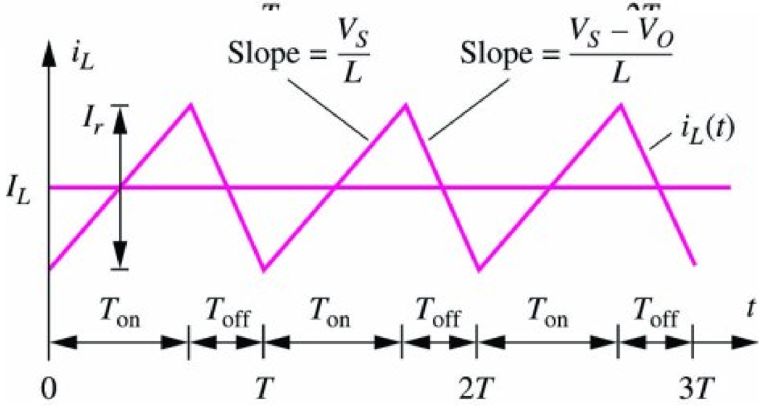
Boost Converter



Switching





When switch is closed, diode is off.
At the end of T_{on}:

$$i_L(T_{on}) = i_L(0^+) + \frac{1}{L}\int_0^{T_{on}} V_S dt = i_L(0^+) + \frac{V_S}{L} T_{on}$$

When switch is open, diode is on.
Assuming constant v_O, at the end of T_{off}:

$$i_L(T) = i_L(0^+) + \frac{V_S}{L} T_{on} + \frac{V_S - V_O}{L} T_{off}$$

In steady state:

$$i_L(T) = i_L(0^+) \qquad \therefore V_O = \frac{V_S}{1-\delta}$$

where $\delta = T_{on}/T$ is **duty cycle**.
Since $0<\delta<1$, $V_o > V_S$

Boost Converter

Ripple current is given by $I_r = \frac{V_S}{L} T_{on}$ or $I_r = \frac{V_o - V_S}{L} T_{off}$

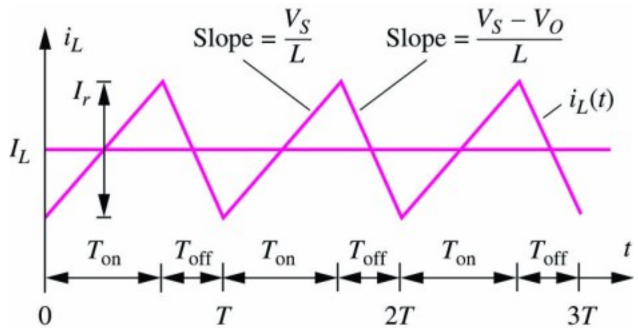
$$\therefore L = \frac{V_S}{I_r} T_{on} = \frac{V_S T}{I_r}\left(\frac{T_{on}}{T}\right) = \frac{V_S}{I_r f}\delta$$

In ideal converter, power delivered to input of converter is same as power delivered to load resistor.

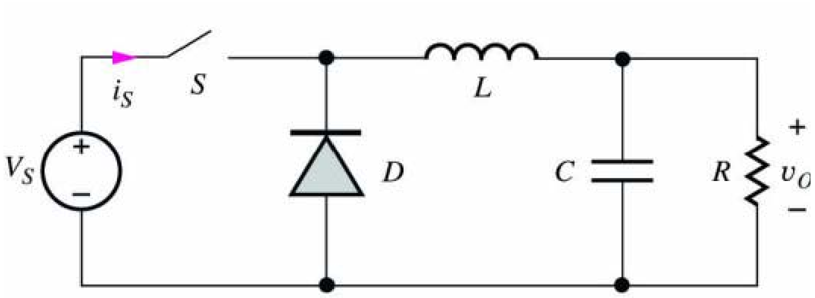
$$V_S I_S = V_o I_o \qquad \text{or} \qquad I_S = I_o \frac{V_o}{V_S} = I_o \frac{T}{T_{off}} = \frac{I_o}{1-\delta}$$

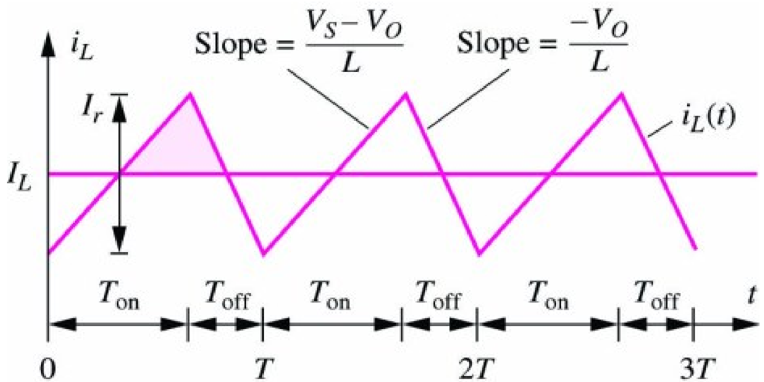
Ripple voltage is given as

$$V_r \approx \frac{I_o}{C} T_{on} = \frac{V_o T_{on}}{RC} = \frac{V_o T}{RC}\left(\frac{T_{on}}{T}\right) = \frac{V_o T}{RC}\delta$$



Buck Converter





When switch is closed, diode is off:

$$i_L(T_{on}) = i_L(0^+) + \int_0^{T_{on}} \frac{V_S - V_o}{L} dt = i_L(0^+) + \frac{V_S - V_o}{L} T_{on}$$

When switch is open, diode is on:

$$i_L(T) = i_L(0^+) + \frac{V_S - V_o}{L} T_{on} - \frac{V_o}{L} T_{off}$$

$$i_L(T) = i_L(0^+)$$

$$\therefore V_o = V_S \frac{T_{on}}{T} = V_S \delta$$

where δ is switch duty cycle.
Since $0<\delta<1$, $V_o < V_S$.

Buck Converter

Ripple current is given by $I_r = \frac{V_o}{L} T_{off}$ or $I_r = \frac{V_S - V_o}{L} T_{on}$

$$\therefore L = \frac{V_o}{I_r} T_{off} = \frac{V_o T}{I_r}\left(\frac{T_{off}}{T}\right) = \frac{V_o}{I_r f}(1-\delta)$$

In ideal converter, power delivered to input of converter is same as power delivered to load resistor.

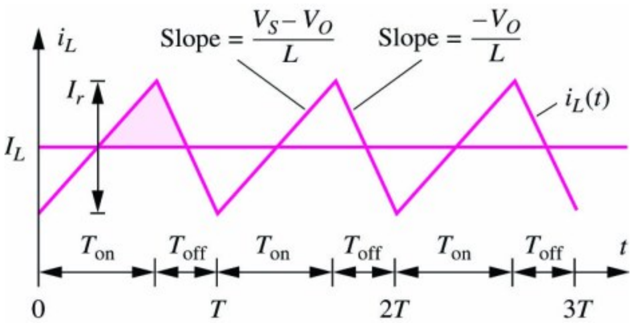
$$\therefore I_S = I_o \frac{V_o}{V_S} = I_o \frac{T_{on}}{T} = I_o \delta$$

Ripple voltage is given as

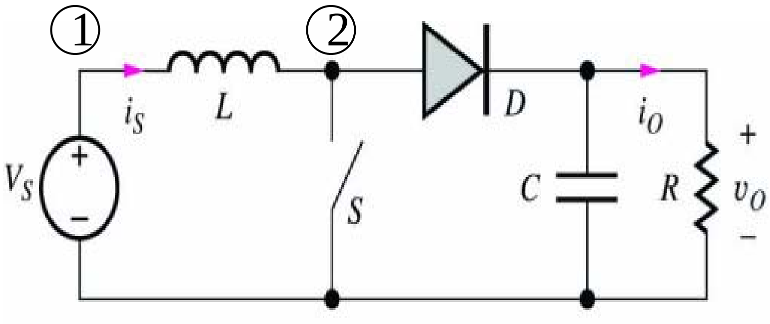
$$V_r = \frac{1}{C}\int_{T_{on}/2}^{T_{on}+T_{off}/2} i_r dt = \frac{\Delta Q}{C}$$

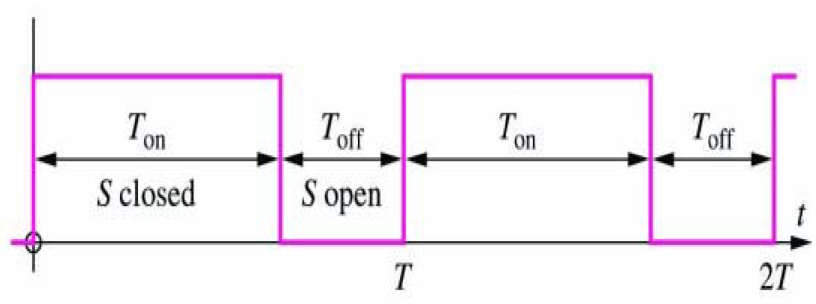
$$\Delta Q = \frac{1}{2}\frac{I_r}{2}\frac{T_{on}+T_{off}}{2} = \frac{I_r T}{8}$$

$$C = \frac{I_r T}{8V_r} = \frac{V_o}{V_r}\frac{T^2}{8L}(1-\delta)$$

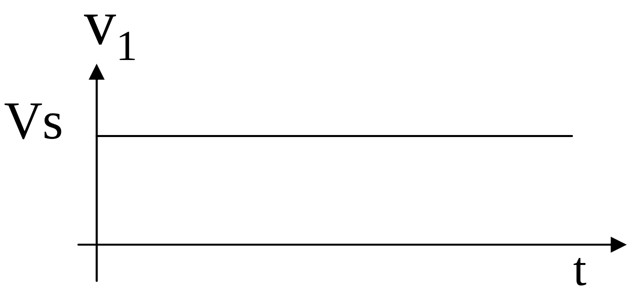


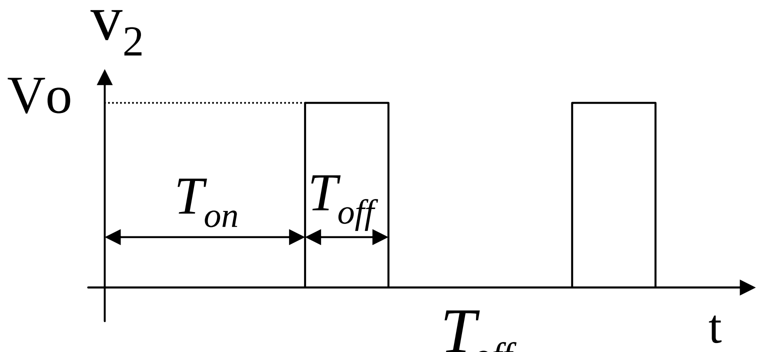
A second analysis approach





In steady state, the average (dc) voltage across the L must be zero, otherwise the inductor current will approach + or – infinity as i_L takes integration of v_L over time.



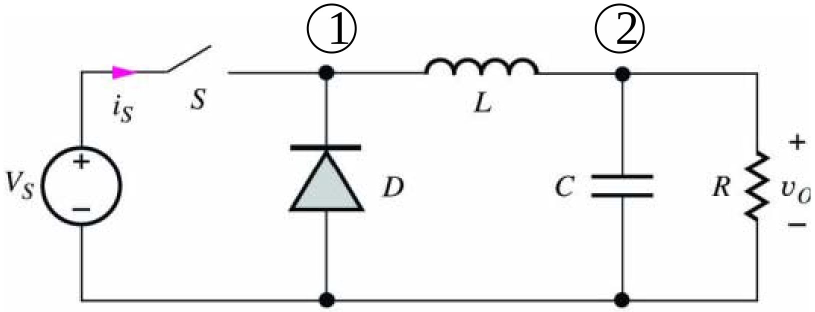


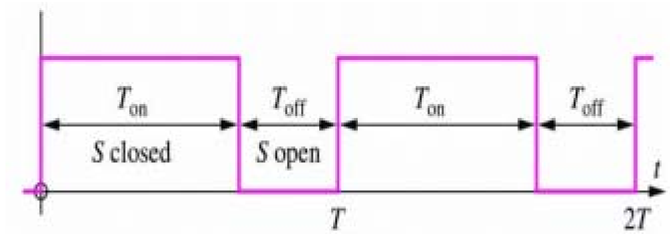
$$< v_1 > = V_S$$

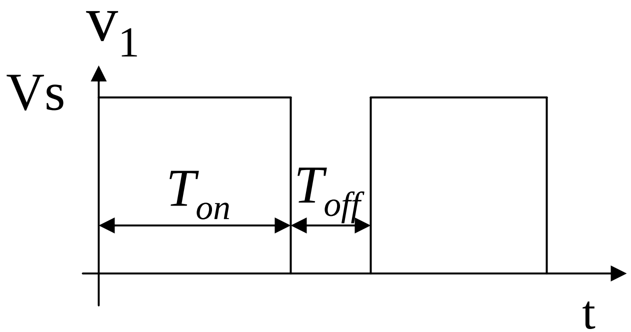
$$< v_2 > = \frac{T_{off}}{T} V_O$$

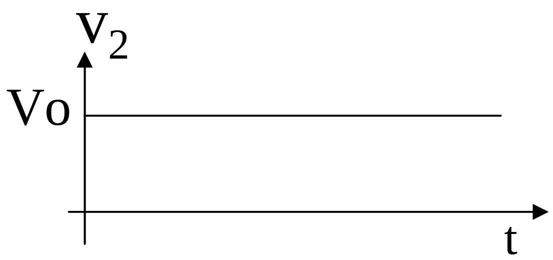
$$\because < v_1 - v_2 > = 0 \quad \therefore V_O = \frac{T}{T_{off}} V_S$$

DC-DC Buck Converter









$$< v_1 > = \frac{T_{on}}{T} V_S$$

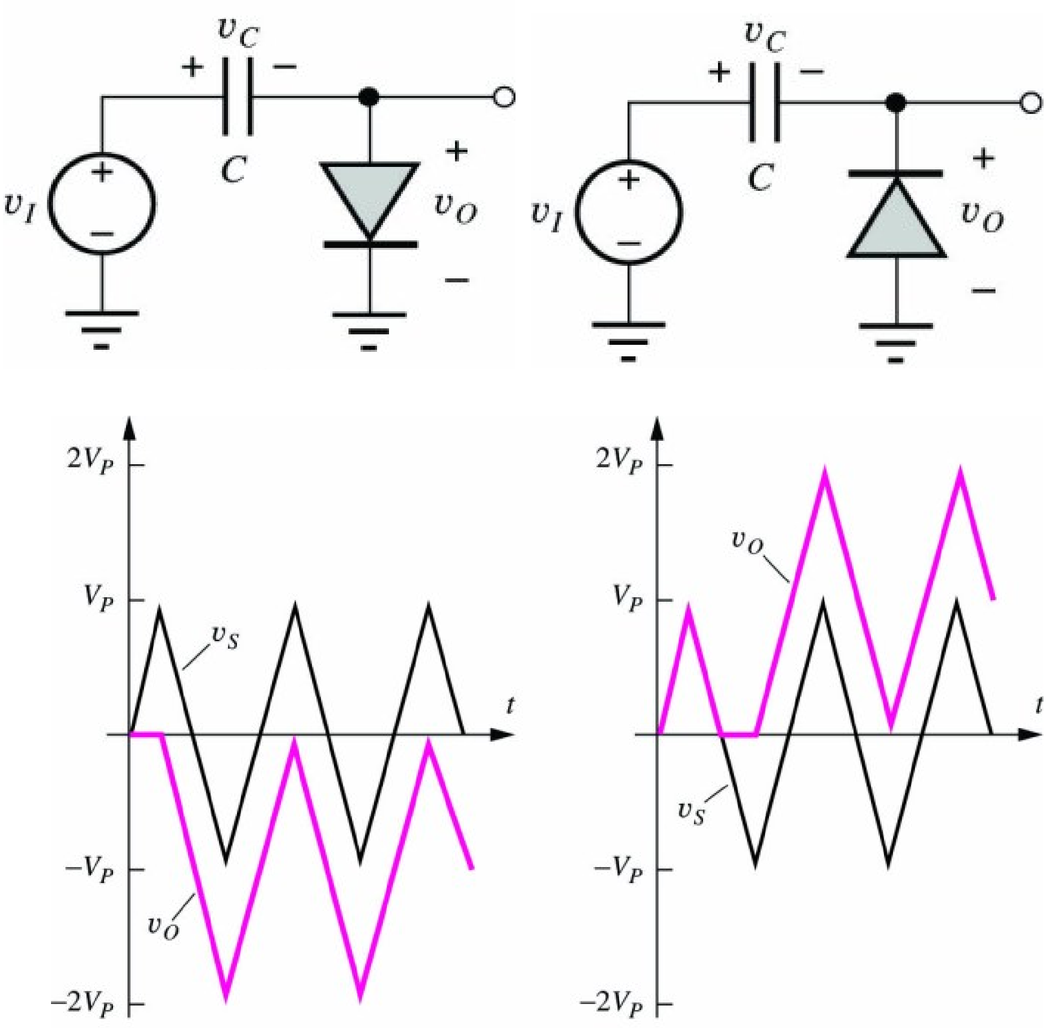
$$< v_2 > = V_O$$

$$\therefore V_O = \frac{T_{on}}{T} V_S$$

Topics to cover …

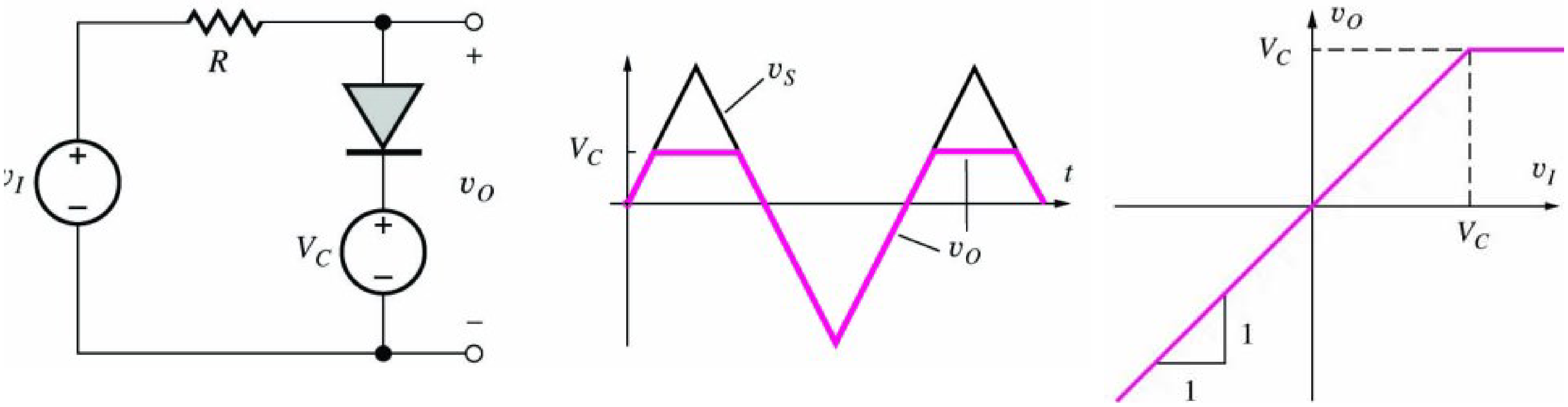
- Voltage Regulation - Zener Diode
- Rectifiers
- DC-to-DC converters
- Wave shaping circuits
- Photodiode and LED

Clamping or DC-Restoring Circuit



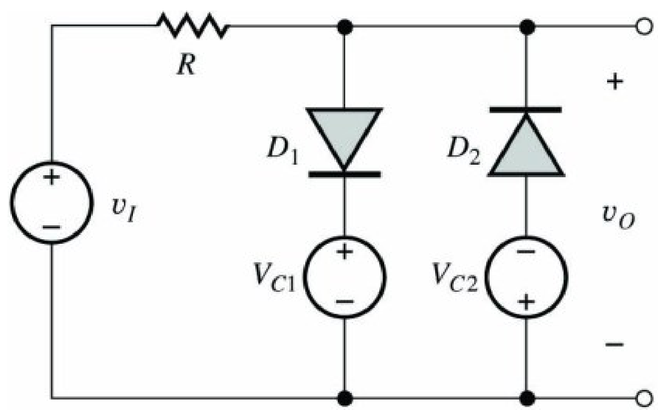
- No DC path between i/p and o/p
- After the initial transient lasting less than one cycle in both circuits, output waveform is an undistorted replica of input
- Both waveforms are clamped to zero
  - Their dc levels are said to be restored
- Clamping level can also be shifted away from zero by adding a voltage source in series with diode

Clipping or Limiting Circuits



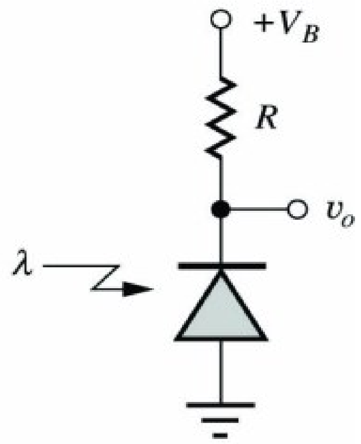
- Have DC path between i/p and o/p

- O/p voltage cannot exceed the clipping level

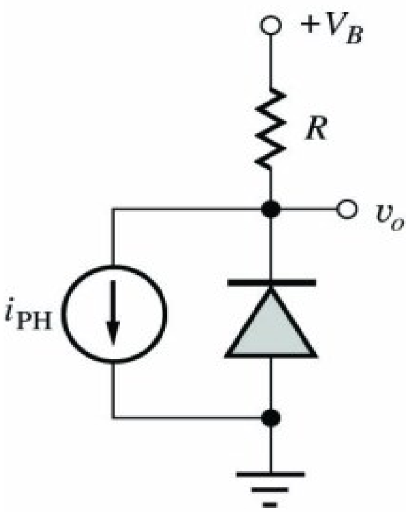


Dual clipping levels

Photodiodes



(a)



(b)

- If depletion region of pn junction diode is illuminated with light with sufficiently high frequency, photons can provide enough energy to cause electrons to jump the semiconductor bandgap to generate electron-hole pairs.

- Photodectector: convert light signal into electrical form. Usually reverse biased to increase the depletion layer width.

- Solar cell: convert solar energy into electrical from.

Light Emitting Diode (LED)

- When electrons and holes recombine, they release energy
- This energy is often released as heat into the lattice, but in some materials, known as *direct bandgap materials*, they release light
- Engineering LEDs can be difficult, but has been done over a wide range of wavelengths