Week 6: Small Signal Analysis for Single Stage BJT amplifiers

Topics to cover …

- Family of single-stage BJT amplifiers

- Small signal analysis
  - Common-Emitter Amplifier

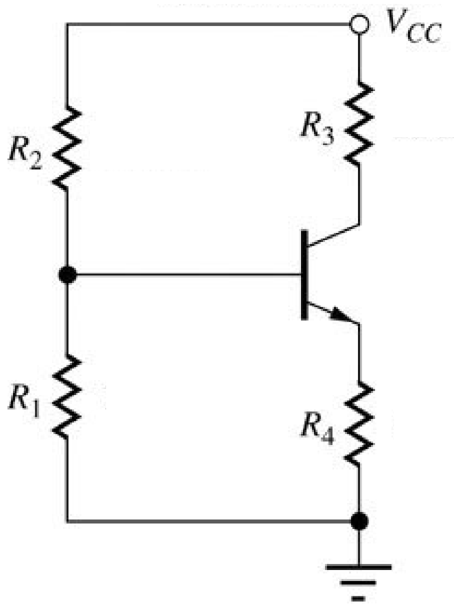
- Common-Emitter Amplifier with Emitter R

- Common-Base Amplifier

Reading Assignment:
Chap 14.1 – 14.5 of Jaeger and Blalock , or
Chap 5.7 of Sedra & Smith

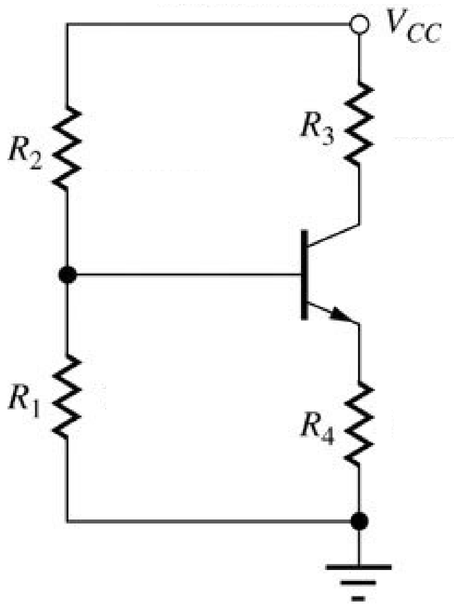
Signal Injection

- Transistor is a VCCS: $i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$



- To cause changes in current, v_{BE} must be changed
  - **Base or emitter** terminals can be used as i/p terminal
  - Collector is not used as an i/p terminal because even if Early voltage is considered, collector voltage has negligible effect on terminal currents

Signal Extraction



- Large changes in i_C or i_E create large voltage drops across collector and emitter resistors
  - **Collector or emitter** can be used as o/p terminal
  - Base terminal is not used as o/p terminal due to small i_B

BJT Amplifier Family

- Constraints for signal injection and extraction yield three families of amplifiers

	Input	Output
– Common-Emitter (C-E):	Base	Collector
– Common-Base (C-B):	Emitter	Collector
– Common-Collector (C-C):	Base	Emitter

  - All circuit examples in the following slides use the four-resistor bias circuits to establish Q-point for the various amplifiers.

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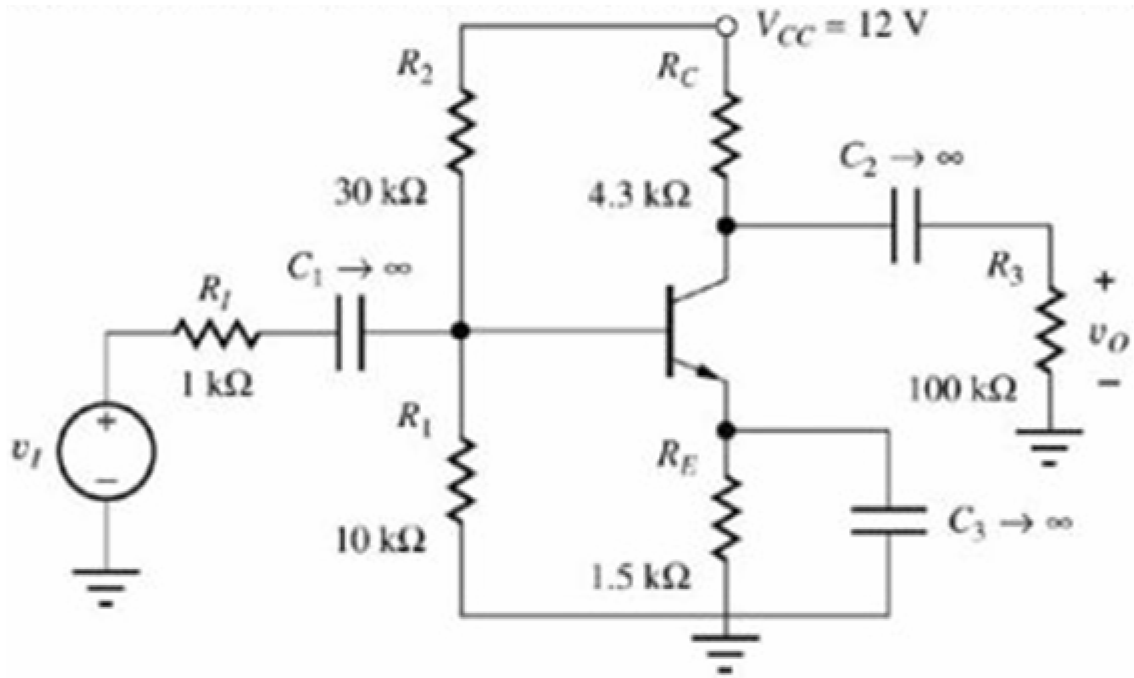
Using Small Signal Models in BJT ckt Analysis

Steps for using small-signal models:

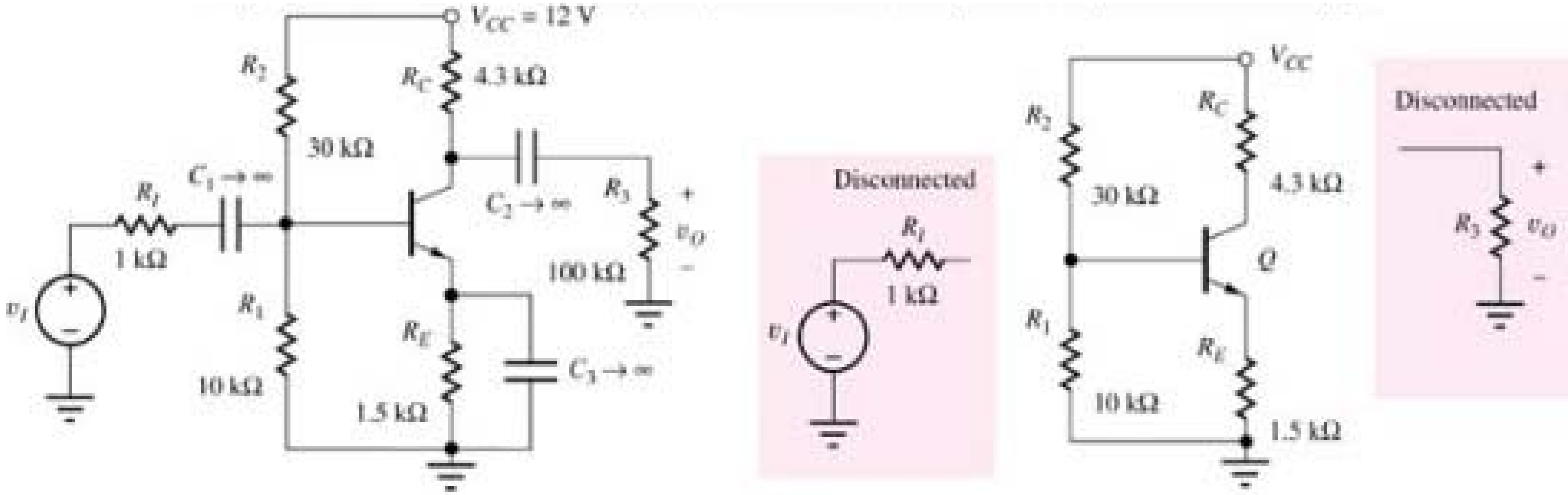
- Determine the DC operating point of the BJT
  - in particular, the collector current
- Calculate small-signal model parameters g_m, r_π, & r_e for this DC operating point
- Eliminate DC sources
  - Replace DC voltage sources with short circuits
  - Replace DC current sources with open circuits
- Replace BJT with an equivalent small-signal model
  - Choose most convenient one depending on surrounding circuitry
- Analyze

Common Emitter Amplifier



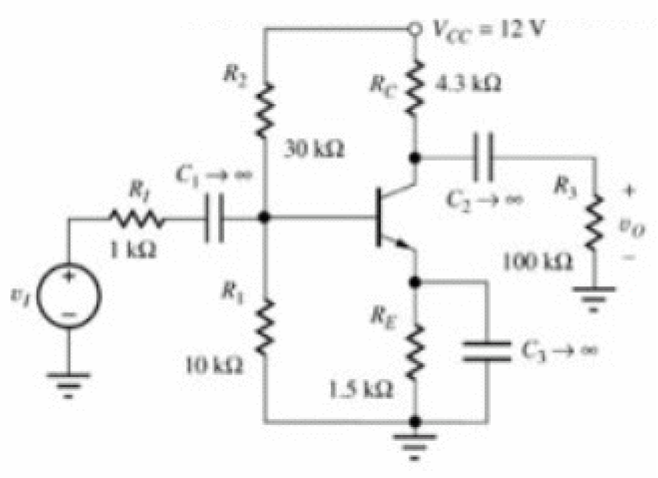
- Biased using the four-resistor network
- AC input is injected to base through a coupling capacitor
- AC output is taken from collector
- Emitter is ac-grounded

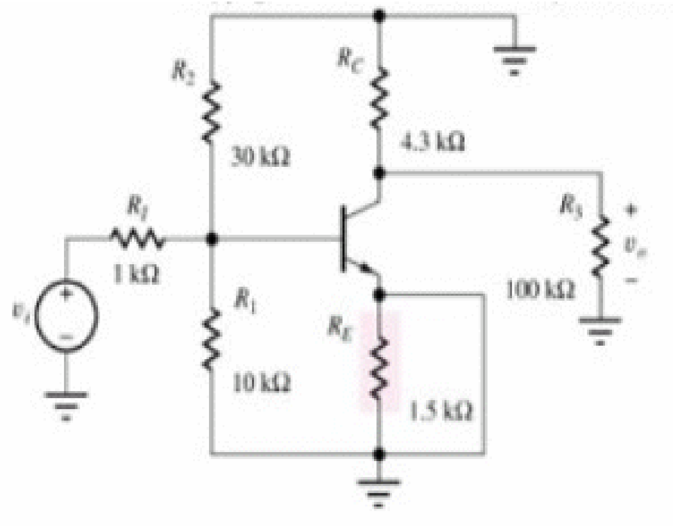
DC operating point



- All capacitors in original amplifier circuits are replaced by open circuits, disconnecting v_I, R_I, and R_3 from circuit
- Q-point can be found from dc equivalent circuit by using DC model for BJT

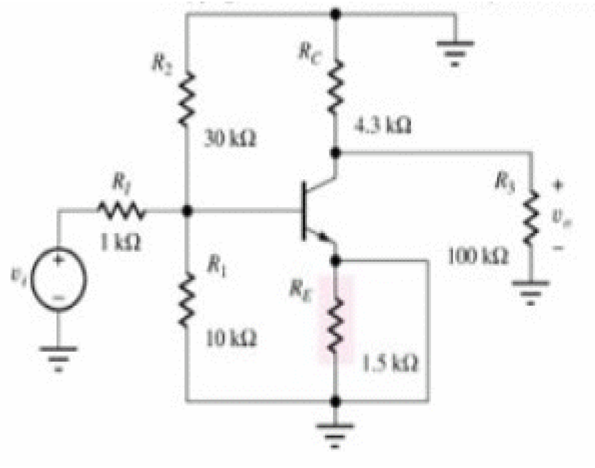
AC Equivalent

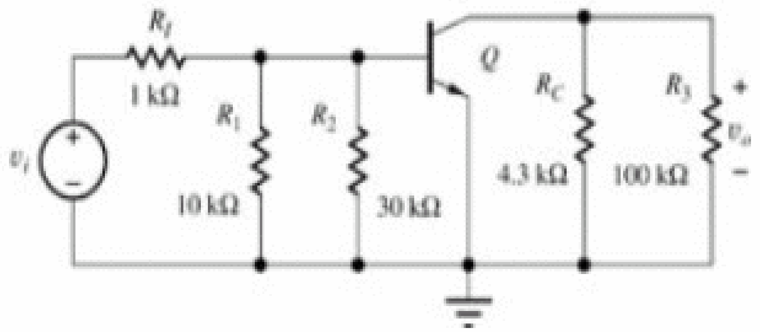


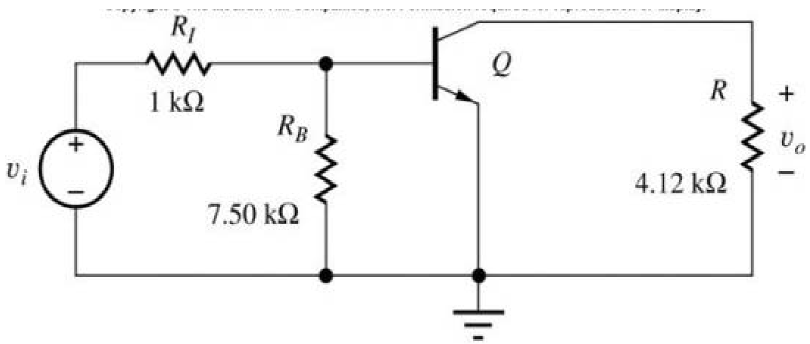


- Replace all ∞ capacitors by short circuits, ∞ inductors by open circuits
- Set all independent DC sources to 0, i.e., replace DC voltage sources by short circuits and DC current sources by open circuits
- Replace transistor by small-signal model
- Use small-signal AC equivalent to analyze AC characteristics of amplifier

Simplified AC Equivalent







$$R_B = R_1 \| R_2 = 10\ \text{k}\Omega \| 30\ \text{k}\Omega$$

$$R = R_C \| R_3 = 4.3\text{k}\Omega \| 100\ \text{k}\Omega$$

Small Signal Performance Parameters

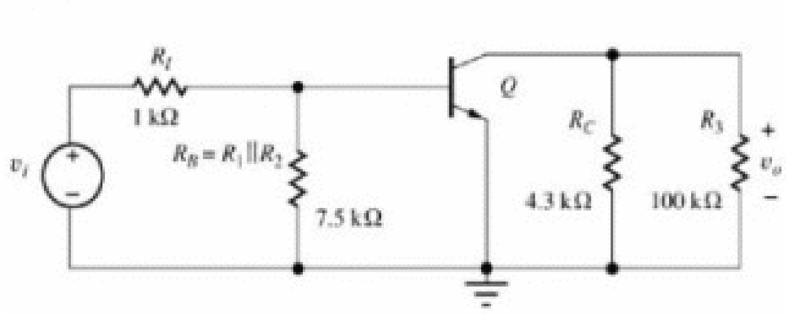
In AC analysis, we are interested to find

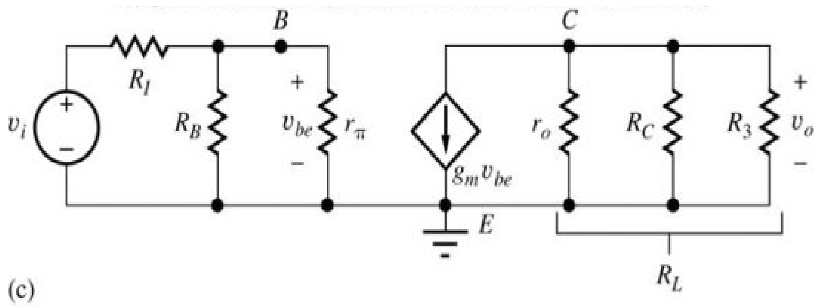
- Voltage gain
- Input resistance
- Output resistance

and sometimes

- Current gain
- Input signal range

Voltage Gain





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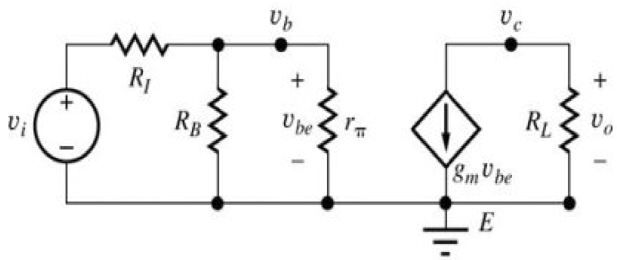


$$R_L = r_o \| R_C \| R_3$$

Terminal voltage gain
between base and collector is:

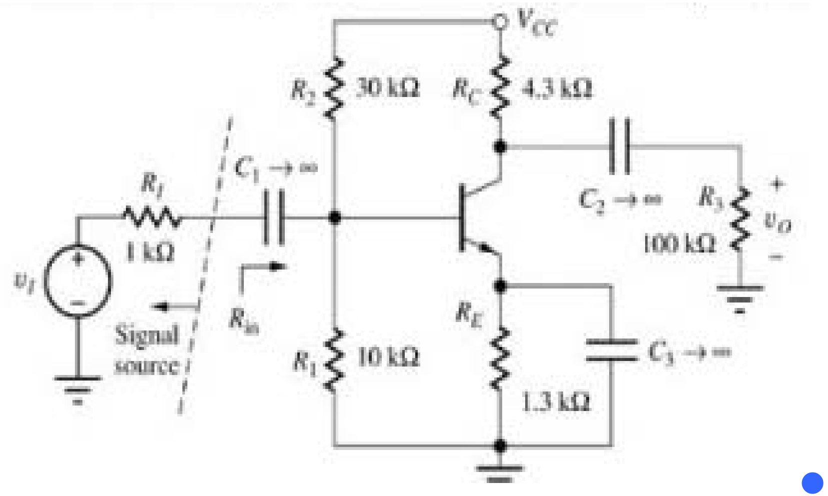
$$A_{vt} = \frac{v_c}{v_b} = \frac{v_o}{v_{be}} = -g_m R_L$$

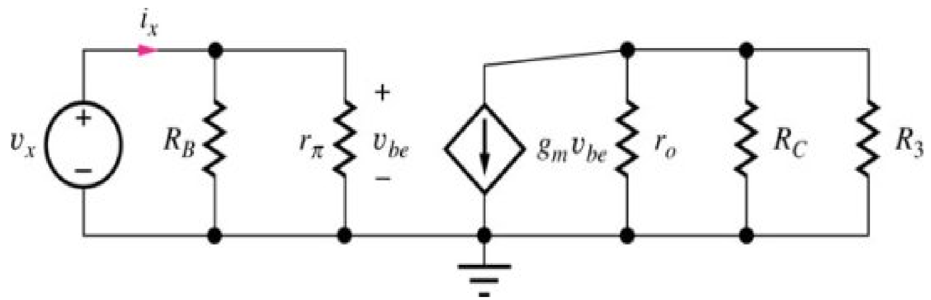
Overall voltage gain from source v_i to output voltage across R_3 is:

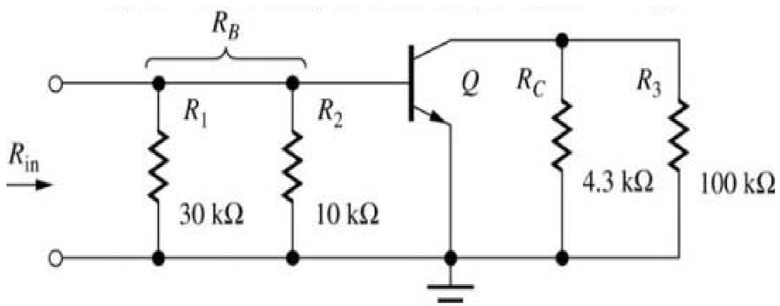
$$A_v = \frac{v_o}{v_i} = \left(\frac{v_o}{v_{be}}\right)\left(\frac{v_{be}}{v_i}\right) = A_{vt}\left(\frac{v_{be}}{v_i}\right)$$

$$\therefore A_v = -g_m R_L \left[\frac{R_B \| r_\pi}{R_I + \left(R_B \| r_\pi\right)}\right]$$

Input Resistance





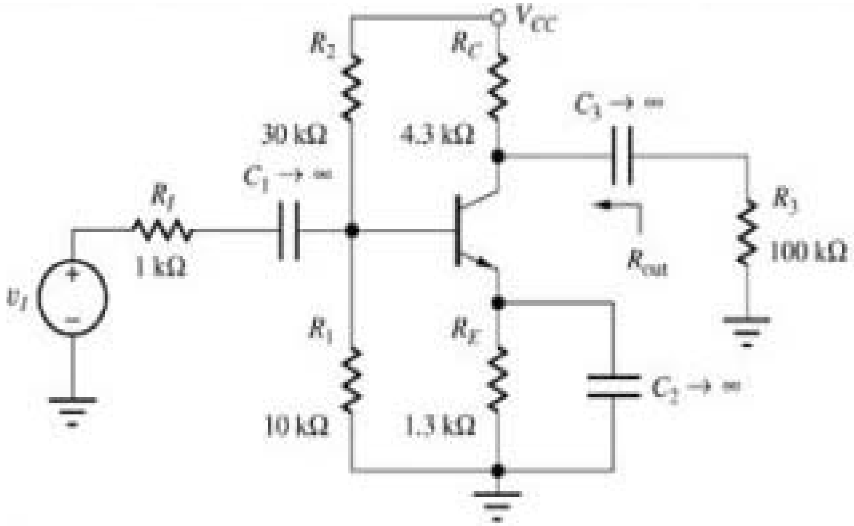


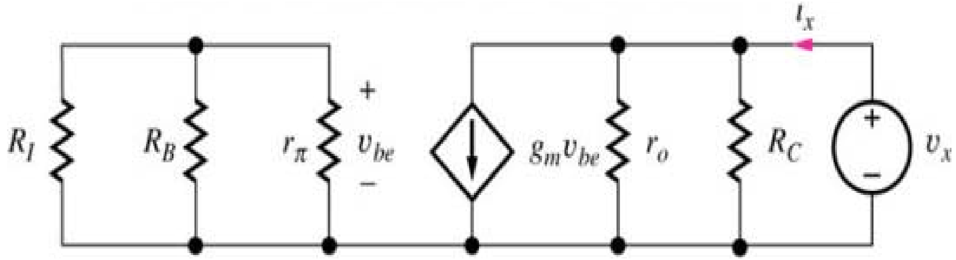
- The total resistance looking into the amplifier at coupling capacitor C_1 represents total resistance of the amplifier presented to signal source

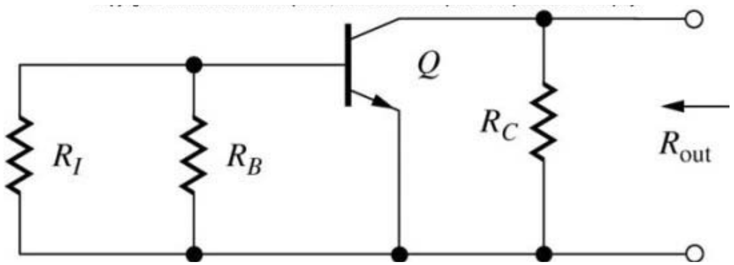
$$v_x = i_x (R_B \| r_\pi)$$

$$R_{in} = \frac{v_x}{i_x} = R_B \| r_\pi = R_1 \| R_2 \| r_\pi$$

Output Resistance





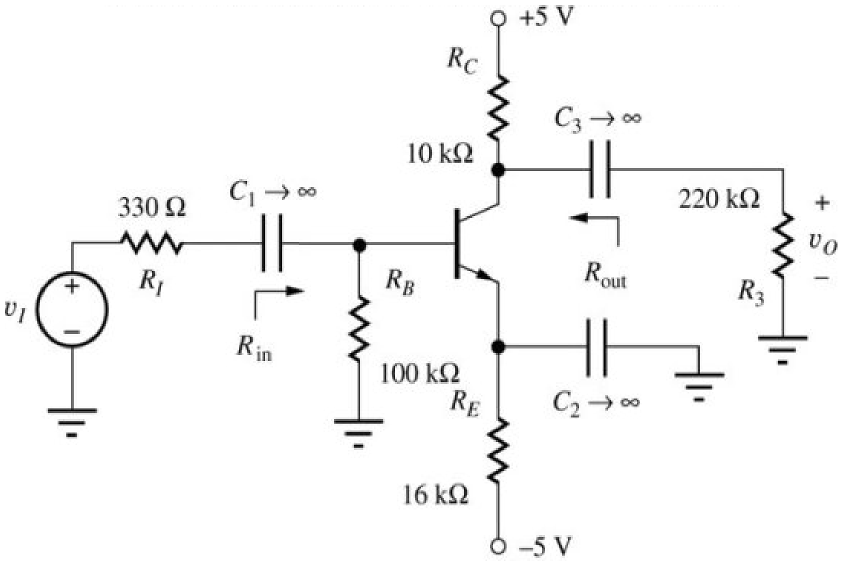


- Output resistance is the total equivalent resistance looking into the output of the amplifier at coupling capacitor C_3
- To find R_{out}, input source is set to 0 and test source is applied at output

$$i_x = \frac{v_x}{R_C} + \frac{v_x}{r_o} + g_m v_{be} \qquad \text{But } v_{be}=0.$$

$$\therefore R_{out} = \frac{v_x}{i_x} = R_C \| r_o \cong R_C \quad \text{As } r_o >> R_C.$$

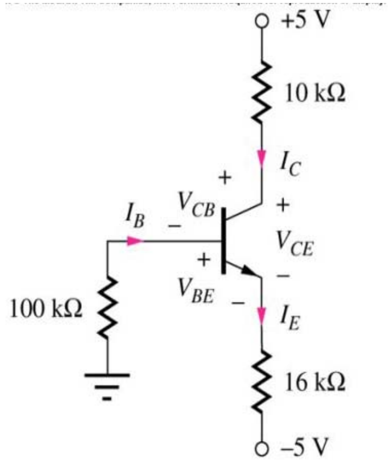
Example 1



- **Problem:** Find voltage gain, input and output resistances.
- **Given data:** β= 65, V_A =50 V
- **Assumptions:** Active-region operation, V_{BE}=0.7 V, small signal operating conditions.

Analysis: To find the Q-point, dc equivalent circuit is constructed.

$$10^5 I_B + V_{BE} + (\beta+1) I_B (1.6\times10^4) = 5$$



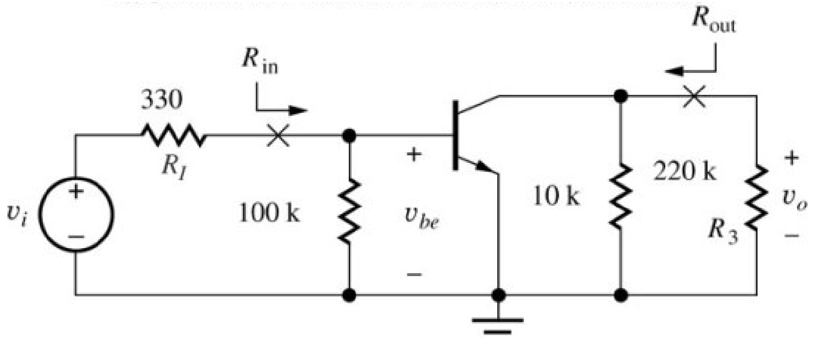
$$\therefore I_B = 3.71\mu A$$
$$I_C = 65 I_B = 241\mu A$$
$$I_E = 66 I_B = 245\mu A$$

$$5 - 10^4 I_C - V_{CE} - (1.6\times10^4) I_E - (-5) = 0$$
$$\therefore V_{CE} = 3.67V$$

Active region of operation is correct.

Example 1 (Cont.)

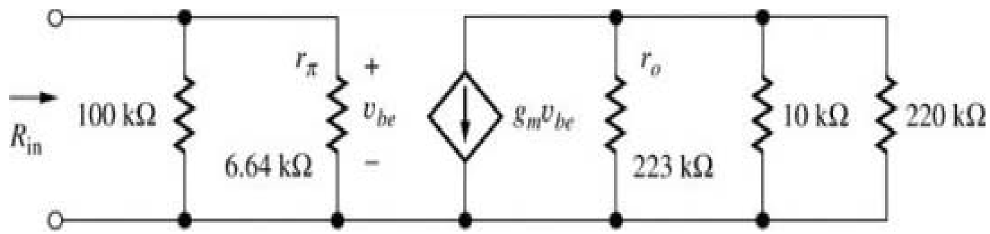
Next we construct the ac equivalent and simplify it



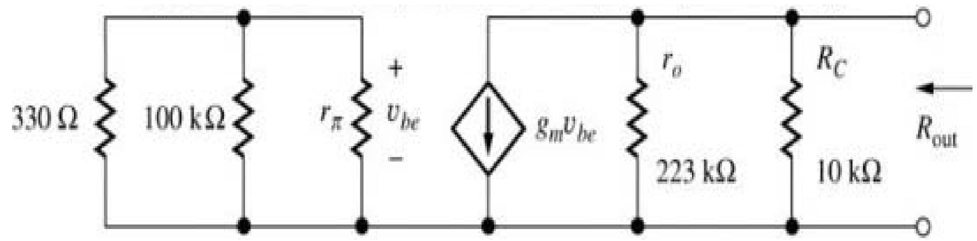
$$g_m = \frac{I_C}{V_T} = 9.64\times10^{-3}\text{S}$$

$$r_\pi = \frac{\beta V_T}{I_C} = 6.64\text{k}\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = 223\text{k}\Omega$$



$$R_{in} = R_B \| r_\pi = 6.23\text{k}\Omega$$



$$R_{out} = R_C \| r_o = 9.57\text{ k}\Omega$$

$$A_v = \frac{v_o}{v_i} = -g_m (R_{out} \| R_3)\left[\frac{R_{in}}{R_I + R_{in}}\right] = -84.0$$

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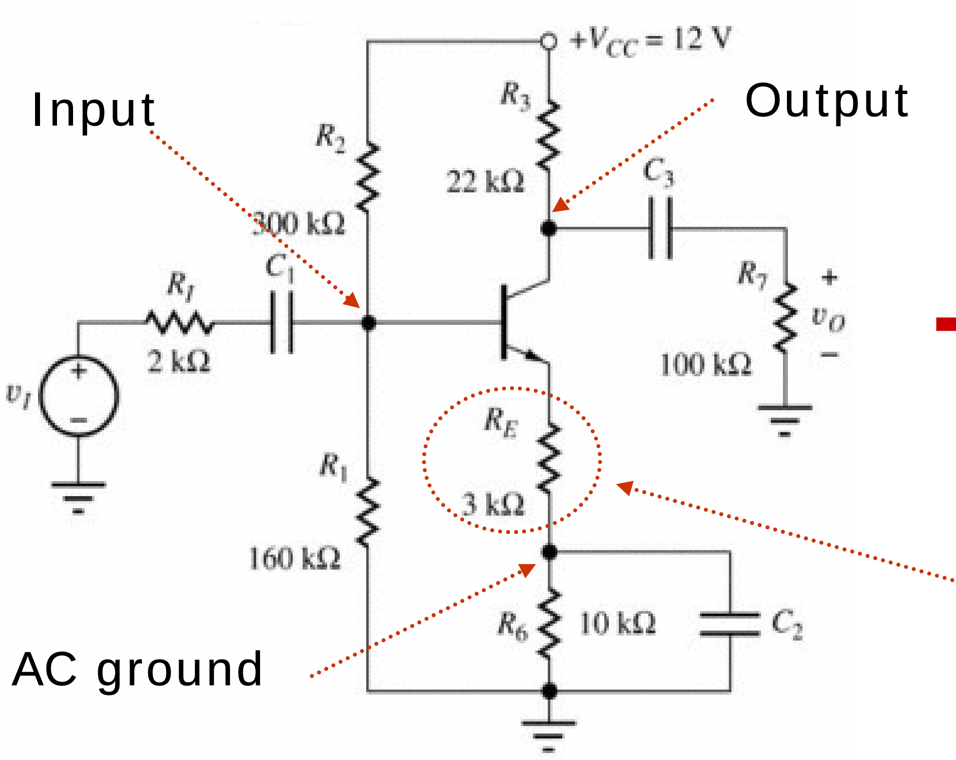
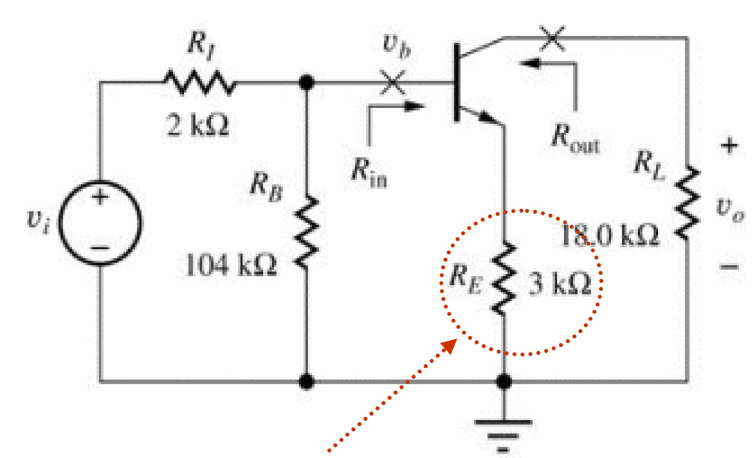
Common-Emitter Amplifier with Emitter R



AC equivalent:

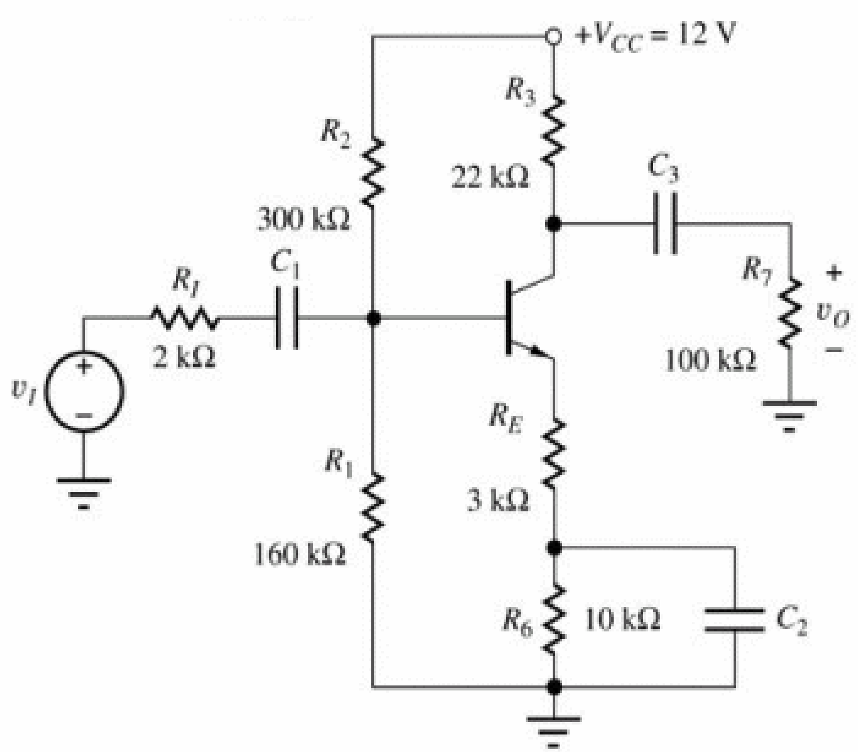


A resistor exists at the emitter of the ac equivalent circuit

Also called ***emitter degenerated*** CE amplifier

R_E provides negative feedback

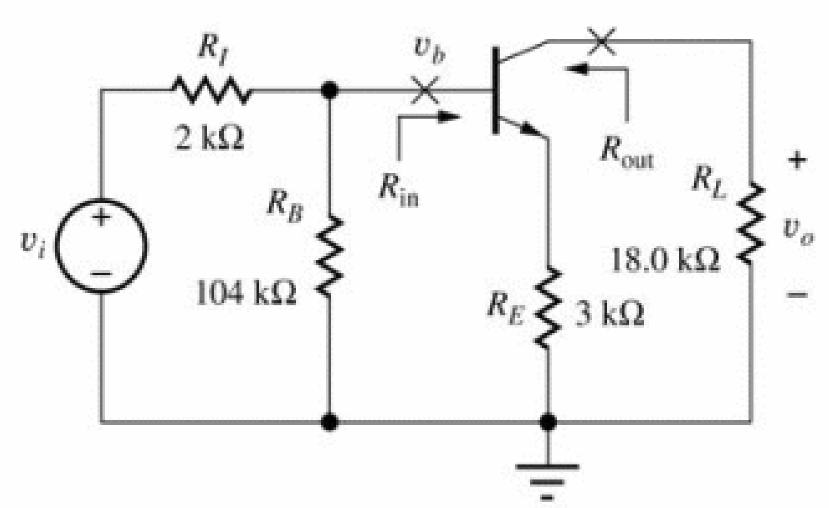
Negative Feedback due to R_E



- Assume i_C ↑ for any reason
- i_E $(=i_C/\alpha)$ ↑
- v_E ↑
- v_{BE} ↓
- i_C ↓ : counter act the original change

- More discussion on negative feedback later in the course

Overall Voltage Gain



Overall voltage gain from v_i to v_o can be expressed as:

$$A_v \equiv \frac{v_o}{v_i} = \left(\frac{v_o}{v_b}\right)\left(\frac{v_b}{v_i}\right)$$

Define ***terminal voltage gain*** as: $A_{vt} \equiv \frac{v_o}{v_b}$

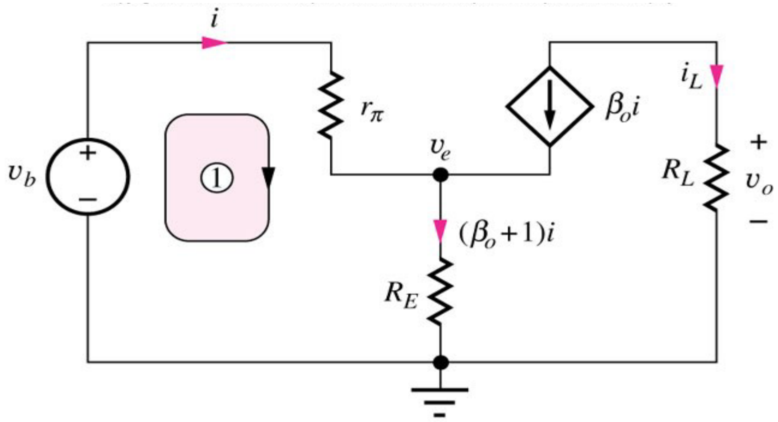
And it can be observed that: $\frac{v_b}{v_i} = \frac{R_B \parallel R_{in}}{R_I + (R_B \parallel R_{in})}$

Thus, the task breaks down into two steps:

- Find the terminal voltage gain
- Find the input resistance R_{in}.

Terminal Voltage Gain

Use hybrid-π model (neglecting r_o):



$$v_b = ir_\pi + (\beta+1)iR_E$$
$$= i\left(r_\pi + (\beta+1)R_E\right)$$

$$v_o = -\beta i R_L$$

$$A_{vt} \equiv \frac{v_o}{v_b} = -\frac{\beta R_L}{r_\pi + (\beta+1)R_E}$$

Using $r_\pi g_m = \beta$, we have

$$A_{vt} = -\frac{g_m r_\pi R_L}{r_\pi + (g_m r_\pi + 1)R_E} \cong -\frac{g_m R_L}{1 + g_m R_E}$$

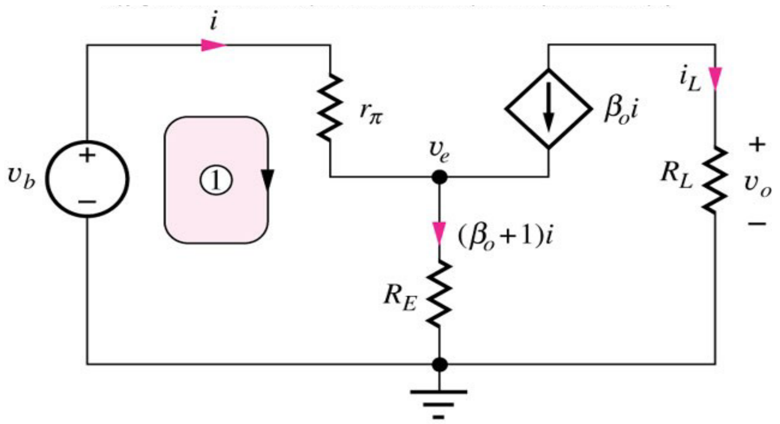
for $g_m r_\pi = \beta >> 1$

Effect of R_E:

- For R_E=0, $A_{vt} \cong -g_m R_L$
  - Upper limit of A_{vt}
- For $g_m R_E$>>1, $A_{vt} = -R_L / R_E$
  - A_{vt} becomes less dependent on g_m which varies widely
- Increasing R_E decreases voltage gain!

Input Resistance

To find R_{in} → to find V_b/i:



$$v_b = i\left(r_\pi + (\beta+1)R_E\right)$$

$$R_{in} = \frac{v_b}{i} = r_\pi + (\beta+1)R_E$$
$$= r_\pi + (g_m r_\pi + 1)R_E$$
$$\cong r_\pi(1 + g_m R_E)$$

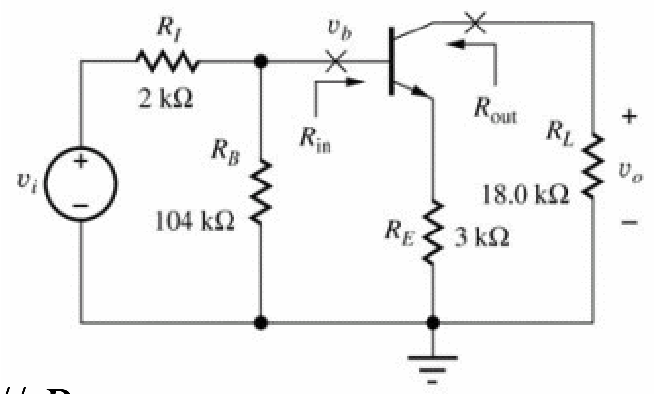
for $g_m r_\pi = \beta >> 1$

- Increasing R_E increases input resistance!

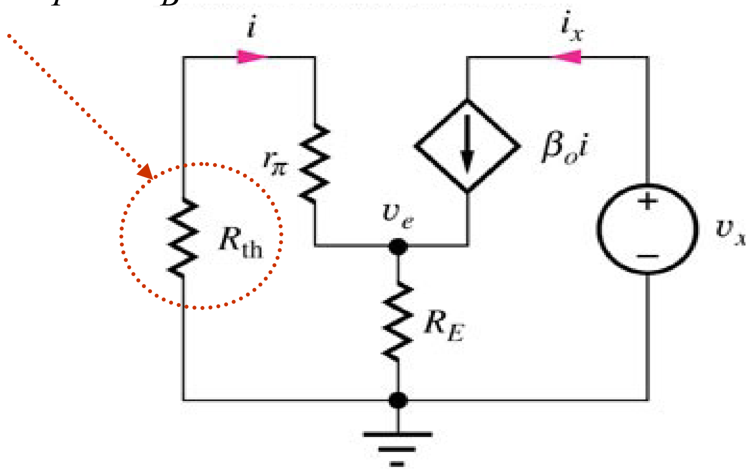
Output Resistance

To find R_{out}:

- Set v_i to zero
- Apply a test source at output
- $R_{out} = v_x/i_x$



$$R_{th} = R_I \,//\, R_B$$



$$v_e = (\beta+1) i R_E$$

KVL at left mesh:

$$i(R_{th} + r_\pi) + v_e = 0$$

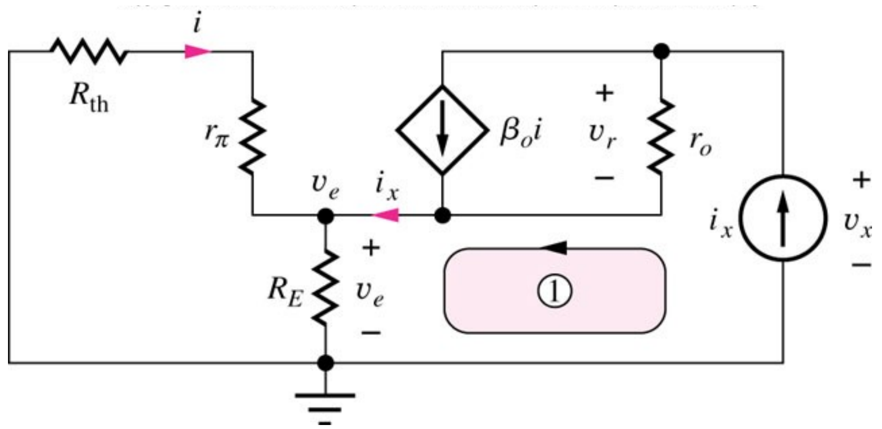
$$\frac{v_e}{(\beta+1)R_E}(R_{th} + r_\pi) + v_e = 0$$

$$\Rightarrow v_e = 0$$

$$\therefore i = 0 \Rightarrow i_x = \beta i = 0 \qquad \therefore R_{out} = \frac{v_x}{i_x} = \infty$$

Output Resistance

Now, we also include r_o in our analysis:



KVL along loop 1:

$$v_x = v_r + v_e = (i_x - \beta i) r_o + v_e$$

Voltage at E:

$$v_e = i_x \left(\left(R_{th} + r_\pi \right) // R_E \right)$$

Current division at Emitter: $i = -i_x \dfrac{R_E}{R_E + R_{th} + r_\pi}$

Put 2nd and 3rd equations into 1st equation:

$$\therefore v_x = r_o \left(i_x + \beta i_x \frac{R_E}{R_E + R_{th} + r_\pi} \right) + i_x \left(\left(R_{th} + r_\pi \right) // R_E \right)$$

$$\therefore R_{out} \equiv \frac{v_x}{i_x} \cong r_o \left(1 + \frac{\beta R_E}{R_E + R_{th} + r_\pi} \right)$$ for large value of r_o

Output Resistance

$$R_{out} \cong r_o \left(1 + \frac{\beta R_E}{R_E + R_{th} + r_\pi} \right)$$

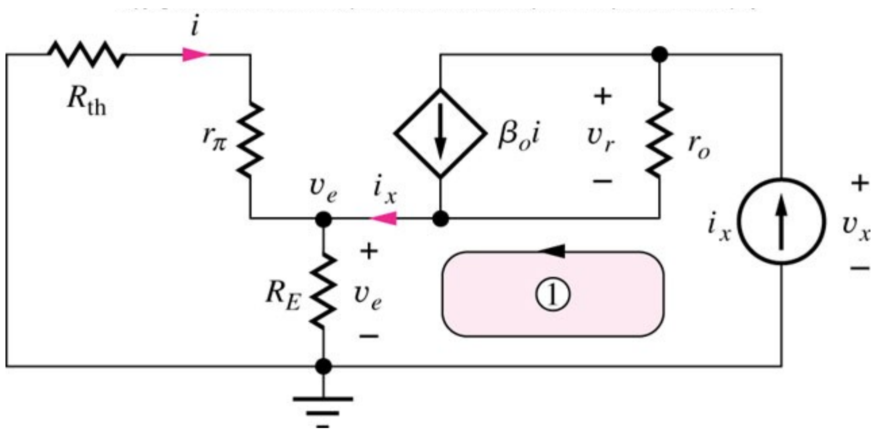
Assuming $(r_\pi + R_E) >> R_{th}$ and $r_o >> R_E$, with $\beta = g_m r_\pi$

$$R_{out} \cong r_o \left(1 + \frac{g_m r_\pi R_E}{R_E + r_\pi} \right) = r_o \left(1 + g_m (r_\pi // R_E) \right)$$

Effect of R_E:

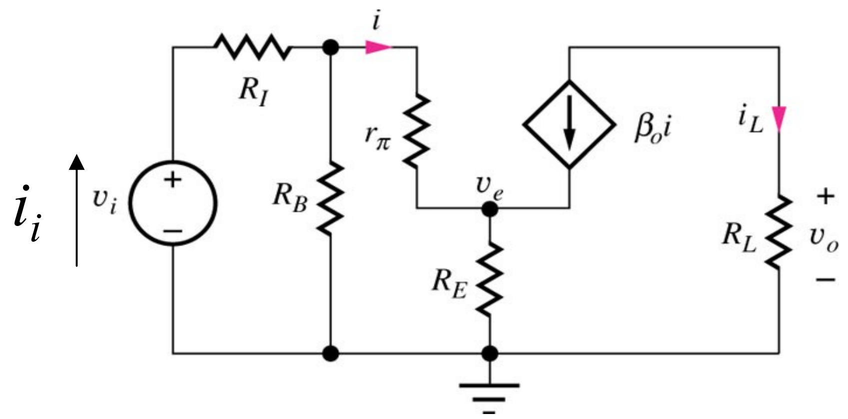
- For $R_E = 0$, $R_{out} = r_o$
- For $R_E = \infty$, upper limit of R_{out} is obtained:

$$R_{out} = (\beta + 1) r_o$$



- Increasing R_E increases R_{out}!

Current Gain



- Terminal current gain: $$A_{it} \equiv \frac{i_L}{i} = -\beta$$

- Overall current gain: $$A_i \equiv \frac{i_L}{i_s} = \frac{i_L}{i}\frac{i}{i_s} = A_{it}\frac{R_B}{R_B + R_{in}}$$

Input Signal Range

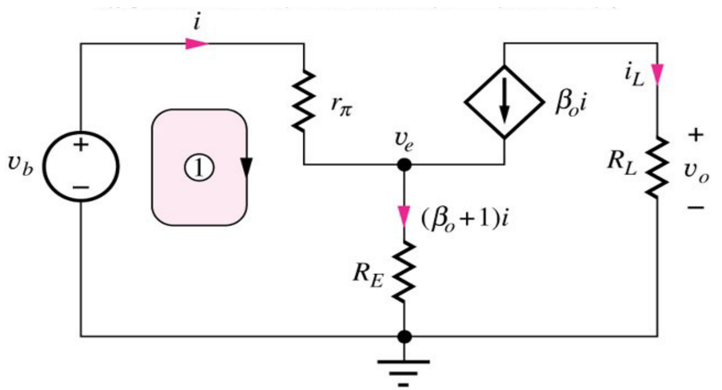
Remember, in deriving the linear small signal model, we assumed $v_{be} << V_T$:

$$i_C = I_C e^{v_{be}/V_T} \cong I_C(1 + v_{be}/V_T)$$

$$= \underbrace{I_C}_{DC} + \underbrace{\frac{I_C}{V_T} v_{be}}_{AC}$$

Typically, v_{be} is required to be less than 5mV!

Input Signal Range



$$v_b = i\left(r_\pi + (\beta+1)R_E\right)$$

$$v_{be} = ir_\pi = r_\pi \frac{v_b}{r_\pi + (\beta+1)R_E} = \frac{v_b}{1 + g_m R_E + R_E / r_\pi}$$

$$\left|v_b\right| \le 0.005\left(1 + g_m\left(R_E + \frac{R_E}{\beta}\right)\right) \cong 0.005(1 + g_m R_E)\text{V}$$

If $g_m R_E >> 1$, v_b can be increased beyond 5 mV limit.

- Condition for $g_m R_E >> 1$:

$$g_m R_E = \frac{I_C R_E}{V_T} \cong \frac{I_E R_E}{V_T} >> 1$$

$$\therefore I_E R_E >> V_T = 0.025\text{V}$$

- Increasing R_E increases input signal range!

Summary of Emitter Degenerated Common-Emitter Amplifier

- *Terminal voltage gain*: Inverting and Large $-\frac{g_m R_L}{1+g_m R_E}$
- *Overall voltage gain*: Inverting and Large $-\frac{g_m R_L}{1+g_m R_E}\left[\frac{R_B // R_{in}}{R_I + R_B // R_{in}}\right]$
- *Input resistance*: Large $r_\pi(1+g_m R_E)$
- *Output resistance*: Large $r_o\left(1+g_m(r_\pi \| R_E)\right)$
- *Terminal Current gain*: Large $-\beta$
- Effects of the resistor at Emitter:
  - Voltage gain decreased, but more stabilized (less sensitive to g_m)
  - Input signal range increased
  - Input and output resistance increased

Topics to cover …

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  - Common-Emitter Amplifier
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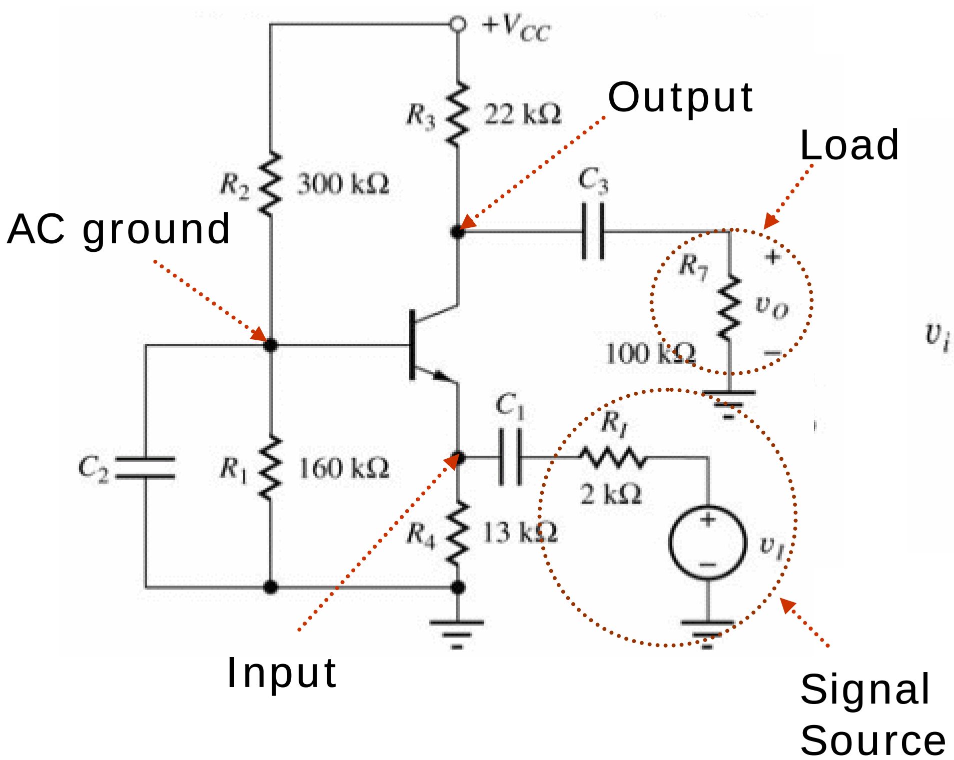
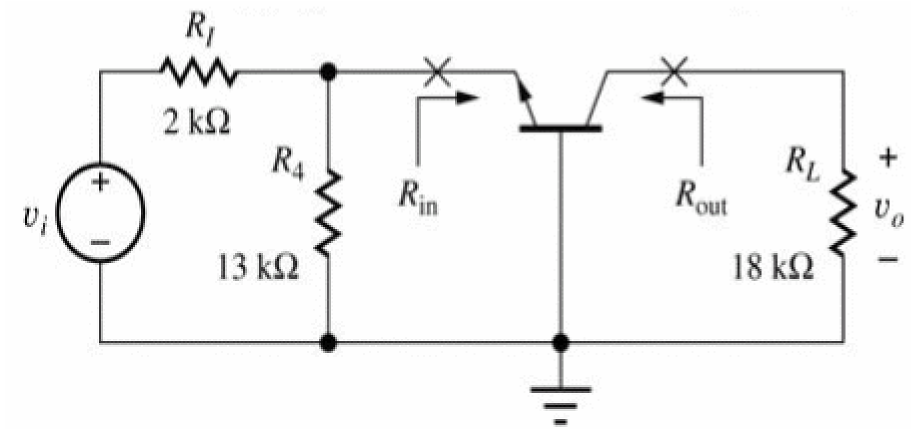
Common-Base Circuits

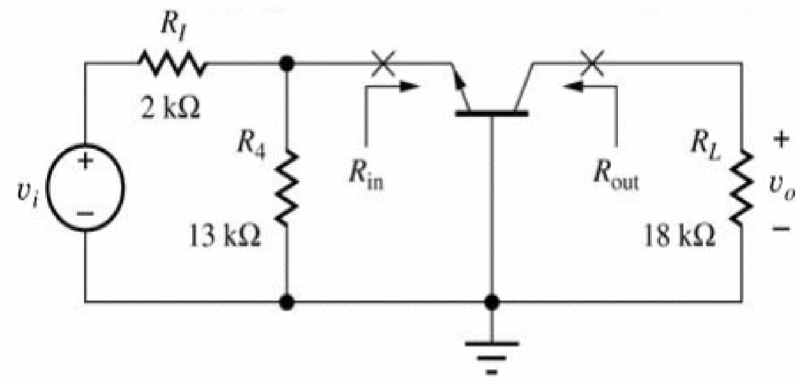


AC/Small-signal equivalent:



$$R_L = R_3 \parallel R_7$$

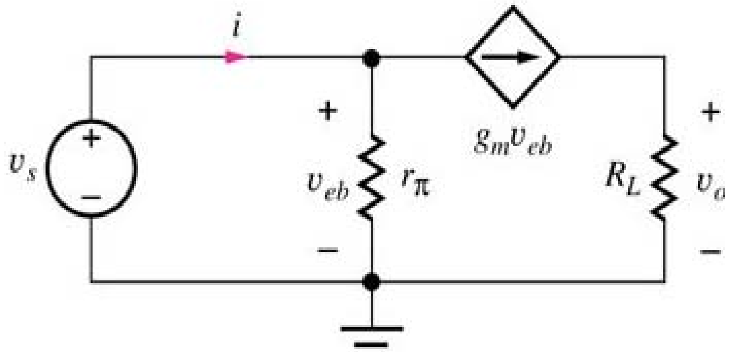
Terminal Voltage Gain



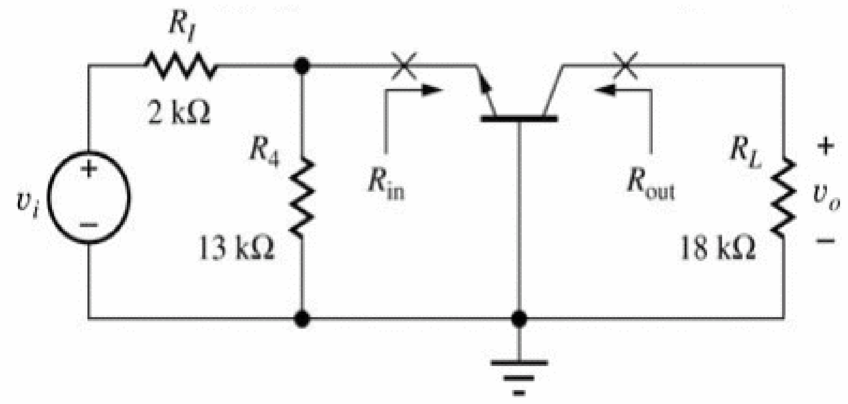
$$A_{vt}^{CB} \equiv \frac{v_o}{v_e} = +g_m R_L$$

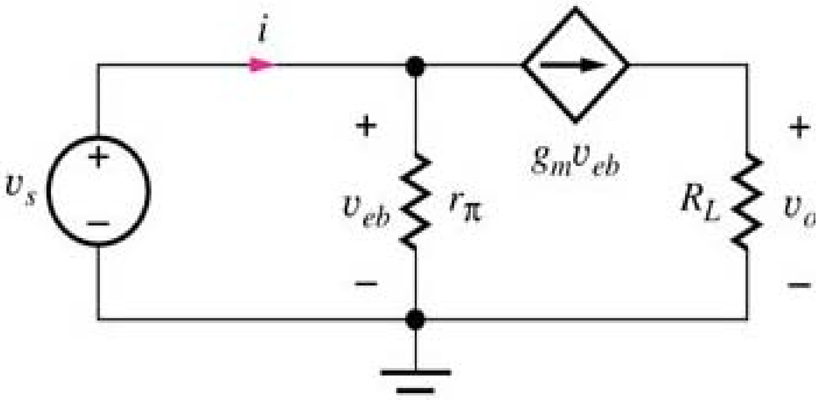
- Non-inverting!
- Magnitude same as the CE amplifier with R_E=0.

Apply test source to input (E)
And use BJT small signal model:



Input Resistance





KCL at emitter:

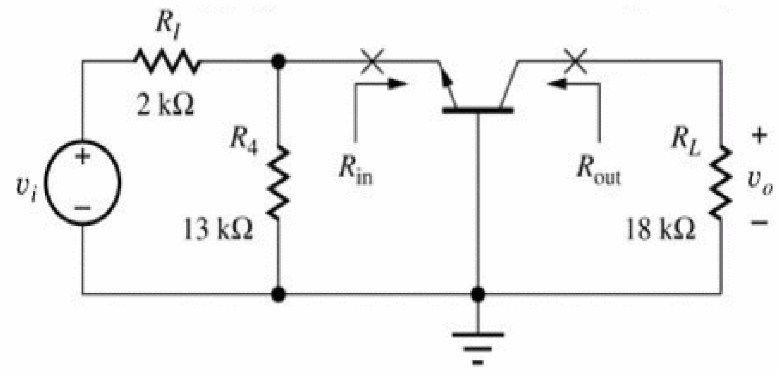
$$i = \frac{v_e}{r_\pi} + g_m v_e$$

$$R_{in}^{CB} = \frac{v_e}{i} = \frac{r_\pi}{r_\pi g_m + 1} = r_\pi // (\frac{1}{g_m}) \cong \frac{1}{g_m}$$

- R_{in} is small (as g_m is usually large)!

$(g_m = I_C / V_T)$

Overall Voltage Gain



Overall voltage gain is

$$A_v^{CB} = \frac{v_o}{v_i} = \left(\frac{v_o}{v_e}\right)\left(\frac{v_e}{v_i}\right) = A_{vt}\left[\frac{R_4 // R_{in}}{R_I + (R_4 // R_{in})}\right]$$

$$= \frac{g_m R_L}{1 + g_m (R_4 // R_I)}\left(\frac{R_4}{R_I + R_4}\right)$$

$$\cong \frac{g_m R_L}{1 + g_m R_I} \quad \text{for} \quad R_4 >> R_I$$

For $g_m R_I << 1$,

$$A_v^{CB} = +g_m R_L$$

This is the upper bound.

For $g_m R_I >> 1$,

$$A_v^{CB} = +\frac{R_L}{R_I}$$

- For large voltage gain, a very small R_I is required!
- Not a good candidate for voltage amplifier

Example

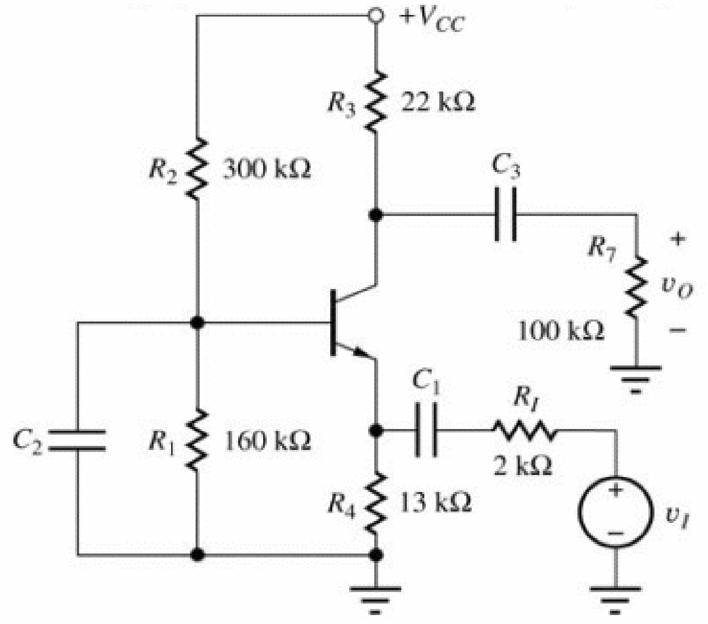
- **Problem:** Find overall voltage gain.
- **Given data:** β=100, Q-point values: I_C=245uA, V_{CE}=3.64V, g_m=9.8mS, r_π=10.2kΩ, r_o=219kΩ.
- **Assumptions:** Small-signal operating conditions.
- **Analysis:**

$$R_{in}^{CB} \cong 1/g_m = 102\Omega$$

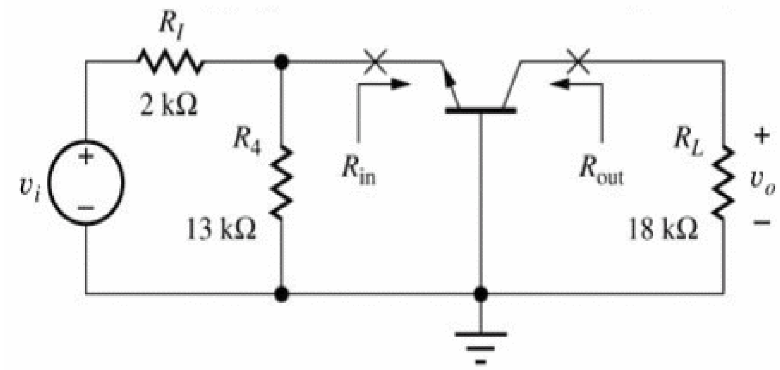
$$R_L = R_3 \| R_7 = 18\text{k}\Omega$$

$$A_{vt}^{CB} = +g_m R_L = 176$$

$$A_v^{CB} = \frac{A_{vt}^{CB}}{1+g_m(R_I \| R_4)}\left(\frac{R_4}{R_I+R_4}\right) = +8.59$$



Input Signal Range



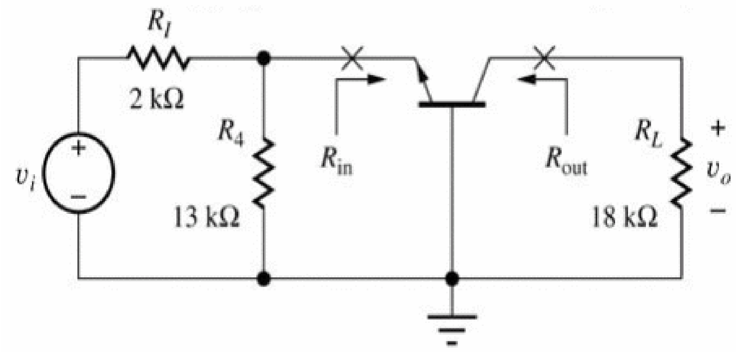
$$v_{eb} = v_i \frac{R_4 // R_{in}}{R_I + R_4 // R_{in}} = \frac{v_i}{1 + g_m (R_I // R_4)} \left(\frac{R_4}{R_I + R_4} \right)$$

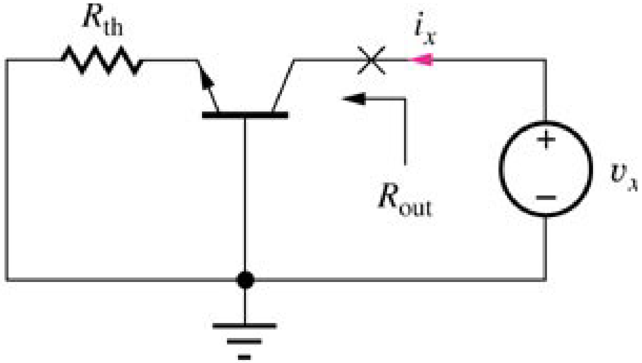
$v_i \cong v_{eb}(1+g_m R_I)$ for $R_4 >> R_I$.

For small-signal operation, $\left| v_b \right| \leq 0.005(1+g_m R_I)\text{V}$

Relative size of g_m and R_I determine signal-handling limit.

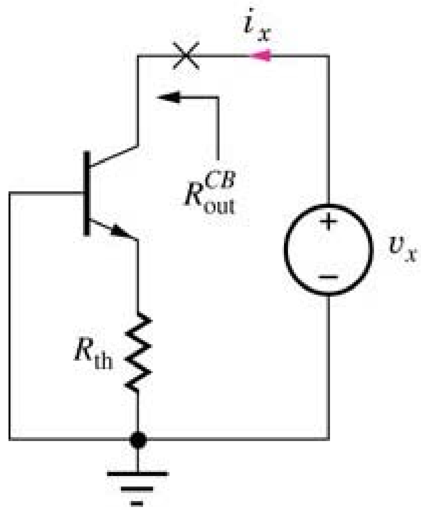
Output Resistance





- Rout here is equivalent to the Rout of CE amplifier with $R_E = R_{th}$ and resistance at base equal to zero.

$$\therefore R_{out}^{CB} = r_o\left(1 + \frac{\beta R_{th}}{R_{th} + r_\pi}\right)$$

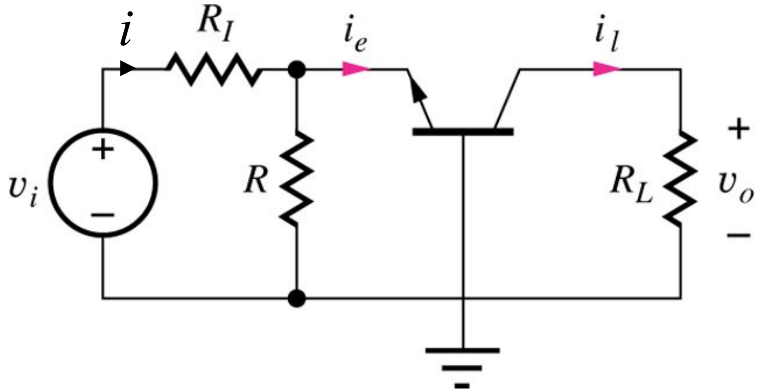


Using $\beta = g_m r_\pi$

$$R_{out}^{CB} \cong r_o\left(1 + g_m (r_\pi // R_{th})\right)$$

- R_{out} is large.

Current Gain



- Terminal current gain: $$A_{it}^{CB} = \frac{i_l}{i_e} = \alpha \cong +1$$

- Current gain from source to load:

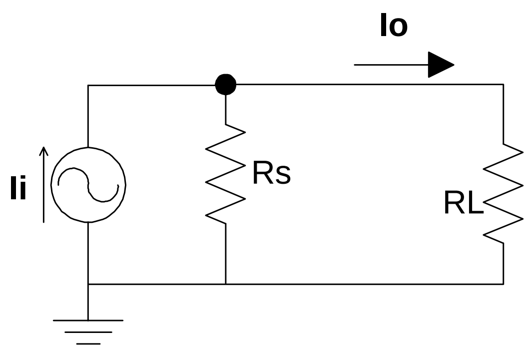
$$A_i^{CB} = \frac{i_l}{i} = \left(\frac{i_l}{i_e}\right)\left(\frac{i_e}{i}\right) = A_{it}\frac{R}{R_{in}+R} \cong A_{it} = 1 \;\; for\; R >> R_{in}$$

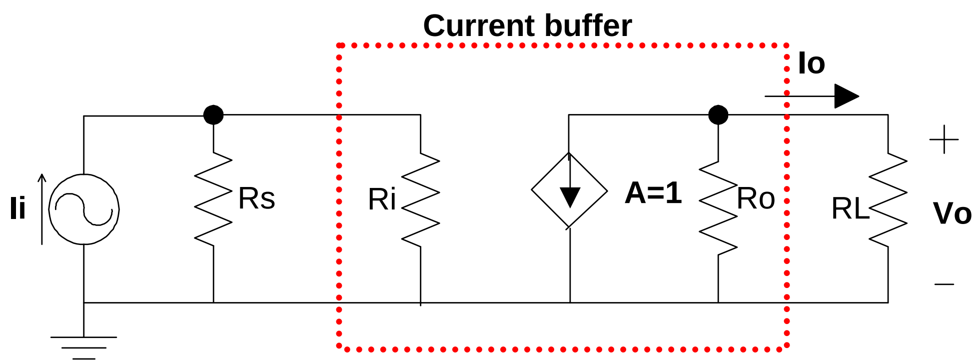
Summary of Common Base Amplifier

- *Terminal voltage gain*: Non-inverting and Large
- *Input resistance*: Low
- *Output resistance*: High
- *Current gain*: close to Unity
- *Input range*: determined by $g_m R_{th}$

- Excellent for use as a current buffer

Current Buffer





- Without a buffer:

$$I_o = \frac{R_S}{R_S + R_L} I_i << I_i \quad \text{for} \quad R_L >> R_s$$

Only a small portion of source current is delivered to load!

- With a buffer:

$$I_o = \left(\frac{R_S}{R_S + R_i} I_i \right) A \frac{R_O}{R_L + R_O}$$

$$\cong AI_i = I_i \ \text{ for } R_i << R_S \text{ and } R_O >> R_L$$

- Requirement of a current buffer:
  - Low input resistance
  - High output resistance
  - Unity current gain