Introduction to AC Machines

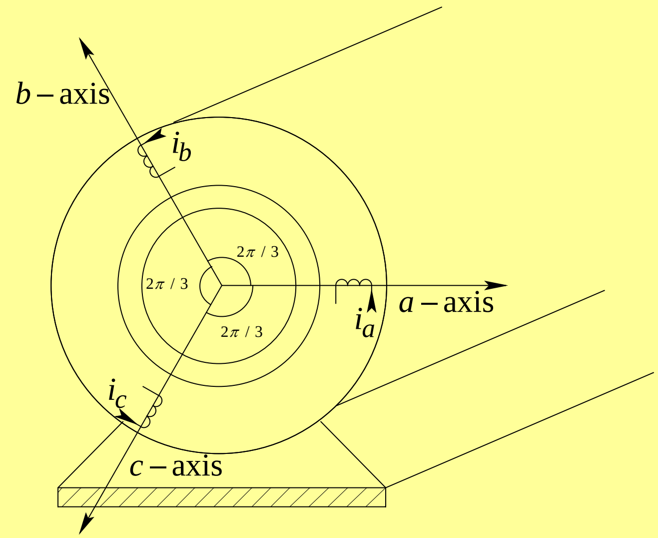
- Introduction
- Sinusoidally-distributed Windings
- Air gap field distribution
- Three-phase windings
- Space vectors to represent sinusoidal distributions

Introduction

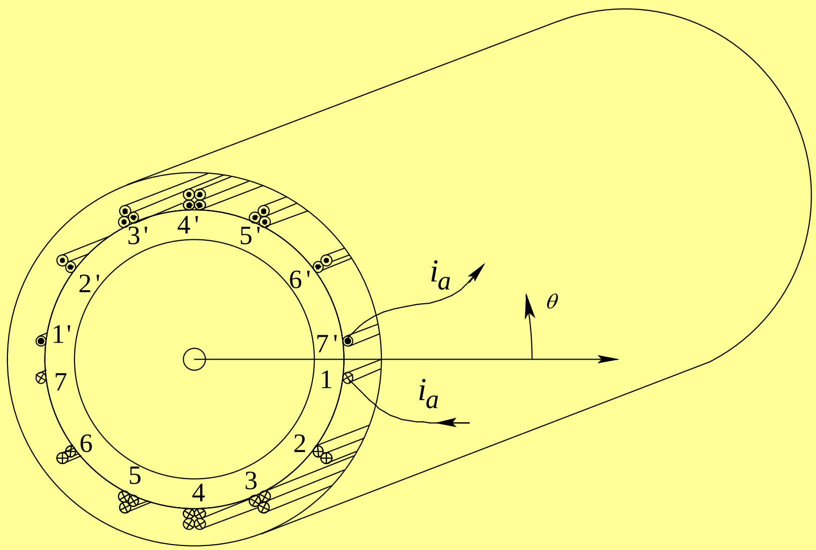
- ☐ Primary AC motor drives
  - ◆ Induction motors
  - ◆ Permanent Magnet Brushless (Synchronous Motors)

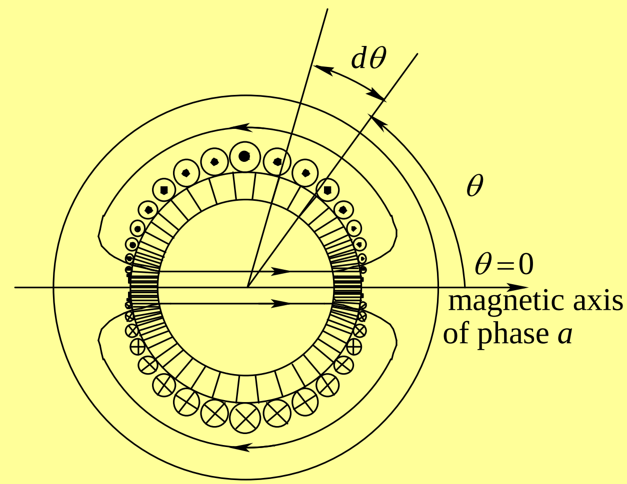
  Both have similar stators but different rotor construction

- ☐ Stator windings produce sinusoidal field distribution



Sinusoidally-distributed Stator Windings





Conductor density $\quad n_s(\theta) = \hat{n}_s \sin(\theta) \quad [no.\ of\ \ conductors / rad] \quad 0 < \theta < \pi$

Total $\quad N_s = \int_0^\pi n_s(\theta)\, d\theta = \int_0^\pi \hat{n}_s \sin(\theta)\, d\theta = 2\hat{n}_s$

$$\Rightarrow \; n_s(\theta) = \frac{N_s}{2} \sin(\theta) \qquad\qquad 0 < \theta < \pi$$

Air-gap Field Distribution

$$H_a(\theta) = -H_a(\theta+\pi)$$

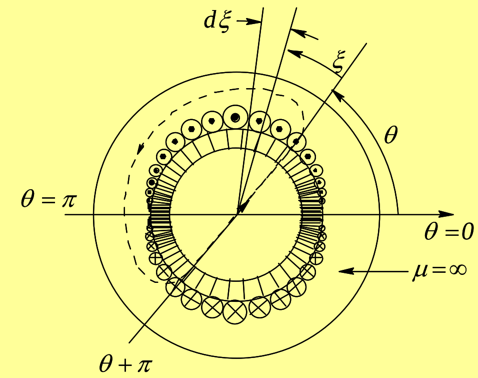
$$H_a(\theta) l_g - H_a(\theta+\pi) l_g = 2H_a(\theta) l_g$$

(negative sign because line of integration points inwards at $\theta+\pi$)

$$2H_a(\theta) l_g = \int_0^{\pi} n_s(\theta+\xi)\, i_a\, d\xi$$

$$2H_a(\theta)\ell_g = \frac{N_s}{2} i_a \int_0^{\pi} \sin(\theta+\xi)\, d\xi = N_s\, i_a \cos(\theta)$$

$$\Rightarrow H_a(\theta) = \frac{N_s}{2\ell_g} i_a \cos(\theta)$$

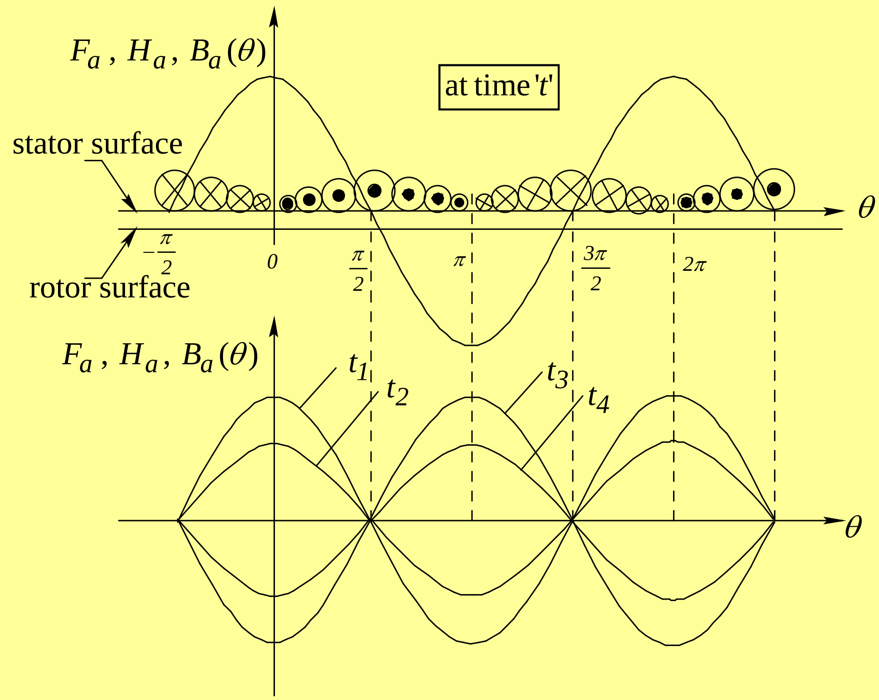


$$H_a(\theta) = \frac{N_s}{2\ell_g} i_a \cos(\theta)$$

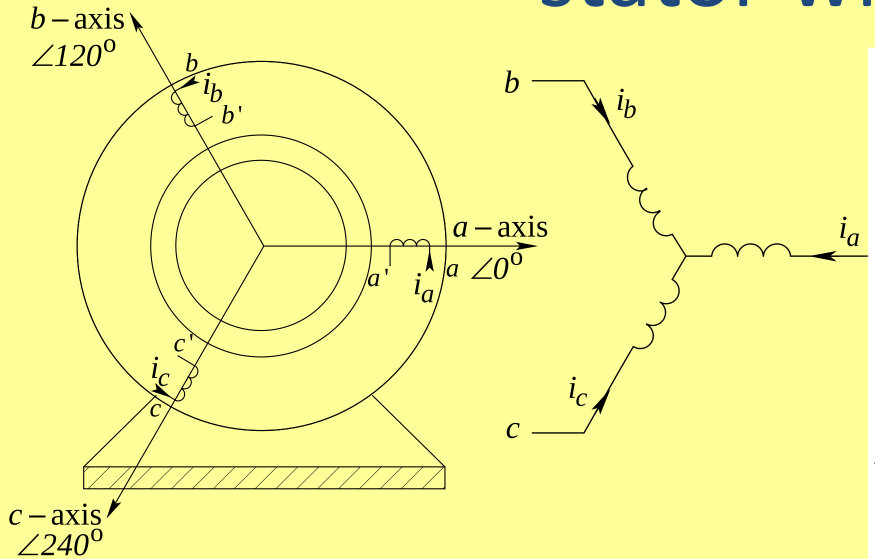
$$B_a(\theta) = \mu_o H_a(\theta) = \left(\frac{\mu_o N_s}{2\ell_g}\right) i_a \cos(\theta)$$

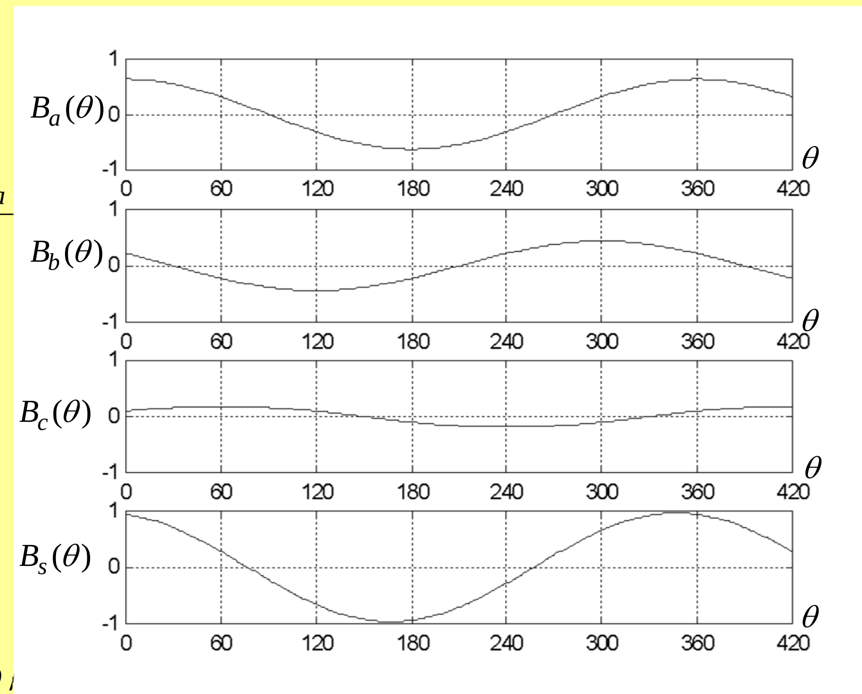
$$F_a(\theta) = \ell_g H_a(\theta) = \frac{N_s}{2} i_a \cos(\theta)$$

Field quantities have different magnitudes and units but same shape



Three-phase sinusoidally-distributed stator windings





- ☐ Example

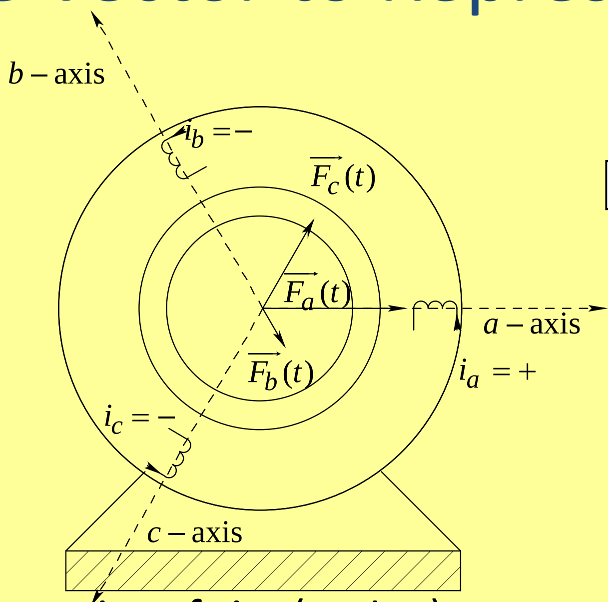
$$B_a(\theta) = \frac{\mu_o N_s i_a}{2\ell_g}\cos\theta = 0.628\cos\theta \ Wb$$

$$B_b(\theta) = -0.440 \times \cos\left(\theta - 120^0\right) \ Wb/m^2$$

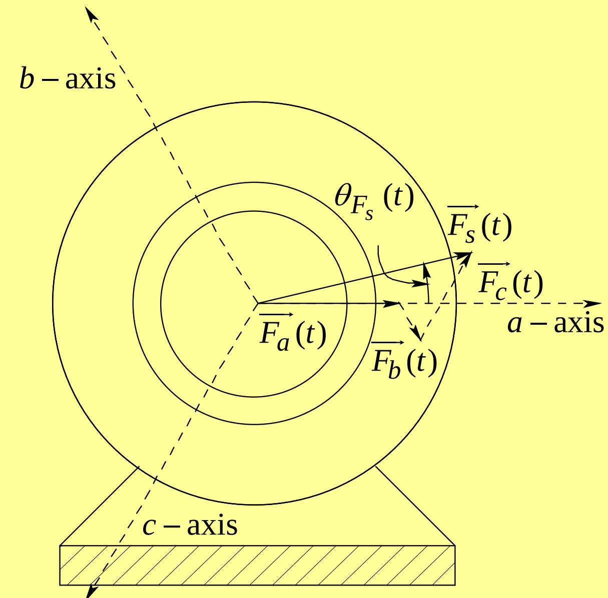
$$B_c(\theta) = -0.188 \times \cos\left(\theta - 240^0\right) \ Wb/m^2$$

$$B_s(\theta) = B_a(\theta) + B_b(\theta) + B_c(\theta) = 0.967\cos\left(\theta - 13.03^\circ\right)$$

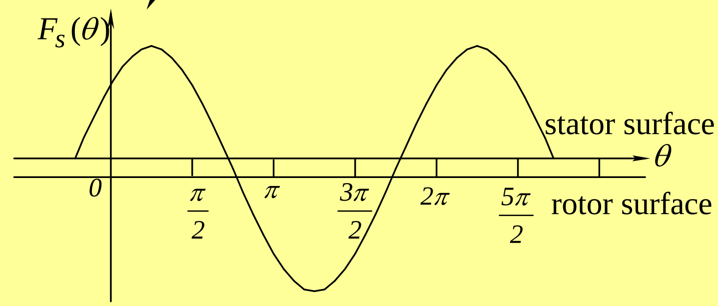
Space Vector to Represent Sinusoidal Distributions



At time ‘t’



- ☐ Properties of sine(cosine)
  - ◆ sum of two sine = sine
  - ◆ integral/derivative of sine = sine
- ☐ Complex number representation



$$F_a(\theta, t) = \frac{N_s}{2} i_a(t)\cos(\theta) \Leftrightarrow \vec{F_a}(t) = \frac{N_s}{2} i_a(t)\angle 0^o$$

Similarly,

$$\vec{F_b}(t) = \frac{N_s}{2} i_b(t)\angle 120^o \qquad \vec{F_c}(t) = \frac{N_s}{2} i_c(t)\angle 240^o$$

And

$$\vec{F_s} = \vec{F_a} + \vec{F_b} + \vec{F_c} = \hat{F}_s \angle \theta_{F_s}$$

- ☐ Similar expressions for *B* and *H*

Example

Three-phase, sinusoidally-distributed stator with $\frac{N_s}{2} = 50$ turns

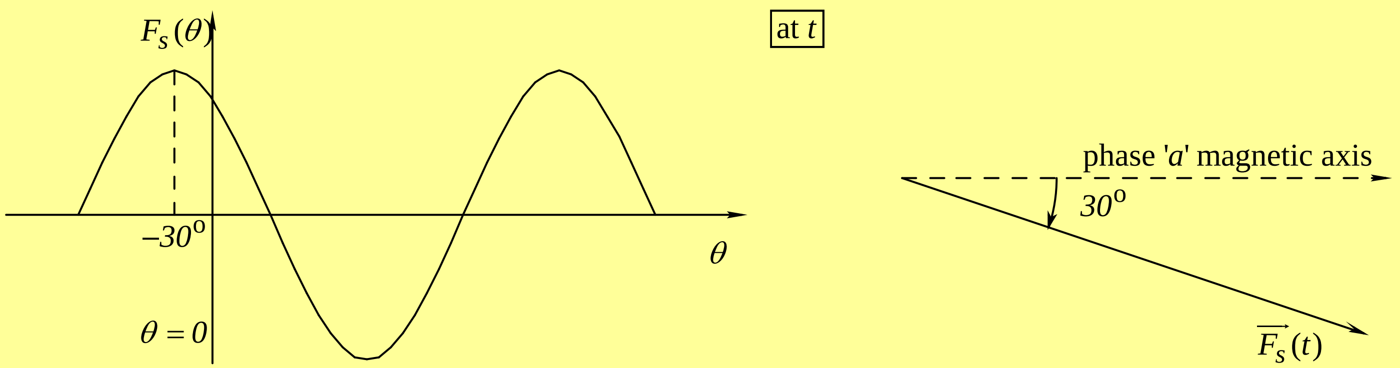
At time t, $i_a = 10A$, $i_b = -10A$ and $i_c = 0A$

Find $\vec{F}$

$$\vec{F}(t) = \frac{N_s}{2}\left(i_a\angle 0^0 + i_b\angle 120^0 + i_c\angle 240^0\right)$$

$$= 50\left\{10 + (-10)\,[\cos 120^0 + j\sin 120^0] + (0)\,[\cos 240^0 + j\sin 240^0]\right\}$$

$$\vec{F}(t) = 50 \times 17.32\angle -30^{\circ} = 866\angle -30^{\circ}\ A\cdot\text{turns}$$



If $l_g = 1.5$ mm and $\mu_m = \infty$, find $\vec{B}(t)$.

$$\vec{B}(t) = 0.73\angle -30^{\circ}\ T$$

Summary

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