Basic Power Flow Equations

$$P_k + jQ_k = \bar{V}_k \bar{I}_k^*$$

$$\bar{I}_k = \sum_{m=1}^{n} Y_{km} \bar{V}_m \qquad Y_{km} = G_{km} + jB_{km} \qquad \bar{V}_k = V_k e^{j\theta_k} \qquad \bar{V}_m = V_m e^{j\theta_m}$$

$$P_k = G_{kk} V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \right)$$

$$Q_k = -B_{kk} V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km} \right)$$

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Equations to be solved for Bus-Voltage Magnitudes and Phase-Angles:

Unknowns:

$$\text{Magnitudes: } n_{PQ}$$

$$\text{Phase-Angles: } n_{PQ} + n_{PV}$$

$$\text{Total: } 2n_{PQ} + n_{PV}$$

Same as the Number of P and Q Equations.

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Gauss-Seidel Procedure:

$$P_k + jQ_k = \bar{V}_k \bar{I}_k^* \qquad \bar{I}_k = \frac{P_k - jQ_k}{\bar{V}_k^*}$$

$$\frac{P_k - jQ_k}{\bar{V}_k^*} = \bar{V}_k Y_{kk} + \sum_{\substack{m \\ m \neq k}} Y_{km} \bar{V}_m$$

$$\bar{V}_k = \frac{P_k - jQ_k}{Y_{kk} \bar{V}_k^*} - \frac{1}{Y_{kk}} \sum_{\substack{m \\ m \neq k}} Y_{km} \bar{V}_m$$

$$Q_k = -\mathrm{Im}\left(\bar{V}_k^* \sum_{m=1}^{n} Y_{km} \bar{V}_m \right)$$

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Newton-Raphson Procedure

$$P_k^{sp} - P_k(V_1,...,V_n,\theta_1,...,\theta_n) = 0 \qquad \text{k} \neq \text{Slack Bus}$$

$$Q_k^{sp} - Q_k(V_1,...,V_n,\theta_1,...,\theta_n) = 0 \qquad \text{k}=PQ \text{ Buses}$$

$$\text{Total of } (2n_{PQ} + n_{PV}) \text{ Equations}$$

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N-R Explanation:

$$c - f(x) = 0 \qquad c - f(x^{(0)} + \Delta x) \simeq 0$$

$$c - [f(x^{(0)}) + \left.\frac{\partial f}{\partial x}\right|_0 \Delta x] = 0 \qquad c - f(x^{(0)}) = \left.\frac{\partial f}{\partial x}\right|_0 \Delta x \qquad \Delta x = \frac{c - f(x^{(0)})}{\left.\frac{\partial f}{\partial x}\right|_0}$$

$$x^{(1)} = x^{(0)} + \Delta x \qquad \Delta x = \frac{c - f(x^{(1)})}{\left.\frac{\partial f}{\partial x}\right|_1} \qquad x^{(2)} = x^{(1)} + \Delta x$$

$$\varepsilon = \left|c - f(x)\right|$$

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N-R Procedure Example:

$$4 - x^2 = 0$$

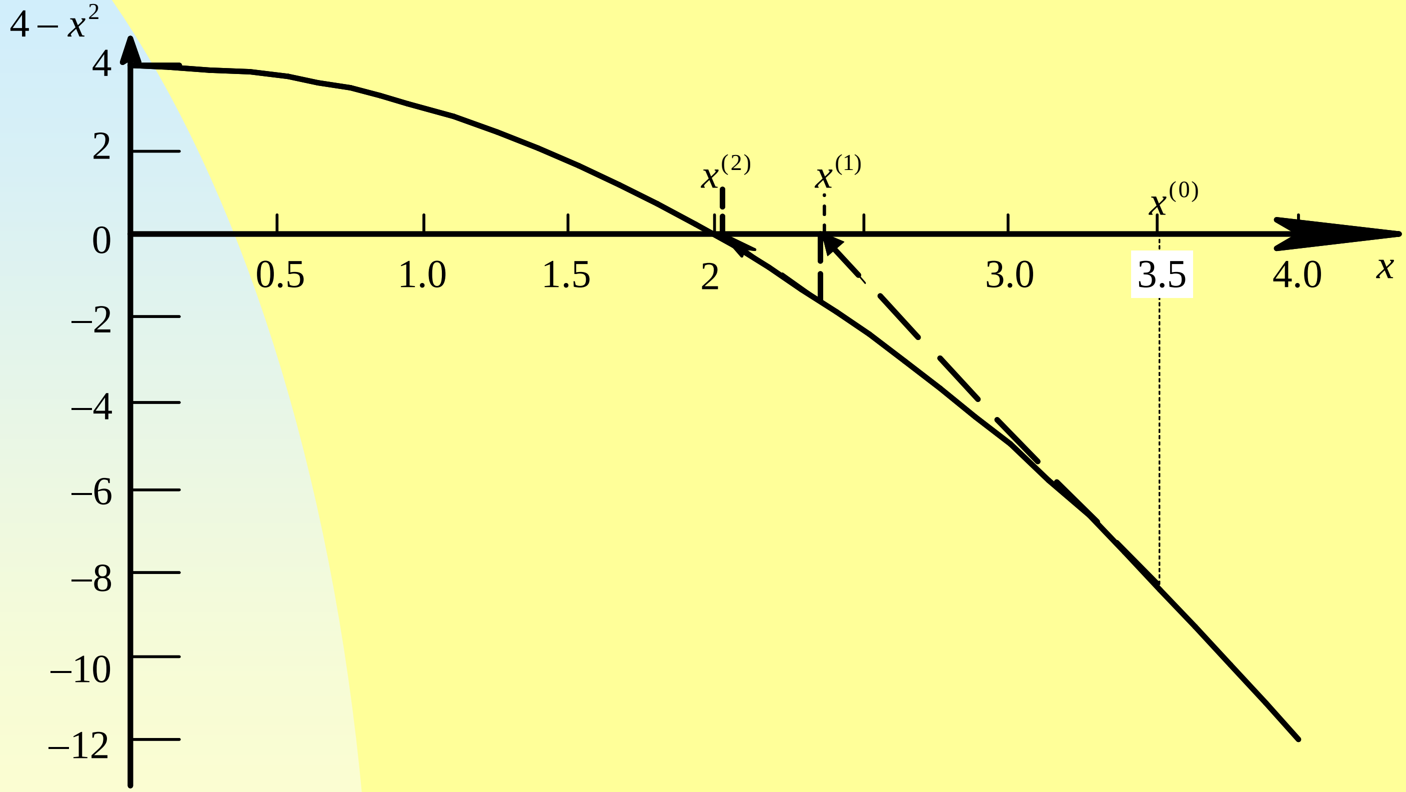

Fig. 5-3 Plot of $4 - x^2$ as a function of x.

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N-R Procedure Applied to n-Bus System:

$$P_k = G_{kk}V_k^2 + V_k \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}\right)$$

$$Q_k = -B_{kk}V_k^2 + V_k \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}\right)$$

$$\frac{\partial P_k}{\partial \theta_k} = V_k \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(-G_{km}\sin\theta_{km} + B_{km}\cos\theta_{km}\right)$$

$$\frac{\partial P_k}{\partial \theta_j} = V_k V_j \left(G_{kj}\sin\theta_{kj} - B_{kj}\cos\theta_{kj}\right) \qquad j \neq k$$

$$\frac{\partial P_k}{\partial V_k} = 2G_{kk}V_k + \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}\right)$$

$$\frac{\partial P_k}{\partial V_j} = V_k \left(G_{kj}\cos\theta_{kj} + B_{kj}\sin\theta_{kj}\right) \qquad j \neq k$$

$$\frac{\partial Q_k}{\partial \theta_k} = V_k \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km}\right)$$

$$\frac{\partial Q_k}{\partial \theta_j} = V_k V_j \left(-G_{kj}\cos\theta_{kj} - B_{kj}\sin\theta_{kj}\right) \qquad j \neq k$$

$$\frac{\partial Q_k}{\partial V_k} = -2B_{kk}V_k + \sum_{\substack{m=1\\m\neq k}}^{n} V_m \left(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km}\right)$$

$$\frac{\partial Q_k}{\partial V_j} = V_k \left(G_{kj}\sin\theta_{kj} - B_{kj}\cos\theta_{kj}\right) \qquad j \neq k$$

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Convergence to the Correct Solution:

$$P_k = G_{kk}V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km}\cos\theta_{km} + B_{km}\sin\theta_{km} \right)$$

$$Q_k = -B_{kk}V_k^2 + V_k \sum_{\substack{m=1 \\ m \neq k}}^{n} V_m \left(G_{km}\sin\theta_{km} - B_{km}\cos\theta_{km} \right)$$

$$\underbrace{\begin{bmatrix} P^{sp} - P \\ Q^{sp} - Q \end{bmatrix}}_{(2n_{PQ}+n_{PV})\times 1} = \underbrace{\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}}_{\underbrace{[J]}_{(2n_{PQ}+n_{PV})\times(2n_{PQ}+n_{PV})}} \underbrace{\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}}_{(2n_{PQ}+n_{PV})\times 1}$$

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = [J]^{-1} \begin{bmatrix} P^{sp} - P \\ Q^{sp} - Q \end{bmatrix}$$

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