# Chapter 9

## Network Theorems




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## Superposition Theorem

- Total current through or voltage across a resistor or branch
  - Determine by adding effects due to each source acting independently
- Replace a voltage source with a short



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# Superposition Theorem

- Replace a current source with an open
- Find results of branches using each source independently
  - Algebraically combine results






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# Superposition Theorem

- Power
  - Not a linear quantity
  - Found by squaring voltage or current
- Theorem does not apply to power
  - To find power using superposition
  - Determine voltage or current
  - Calculate power



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# Thévenin's Theorem

- Lumped linear bilateral network
  - May be reduced to a simplified two-terminal circuit
  - Consists of a single voltage source and series resistance




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# Thévenin's Theorem

- Voltage source
  - Thévenin equivalent voltage, $E_{Th}$.
- Series resistance is Thévenin equivalent resistance, $R_{Th}$

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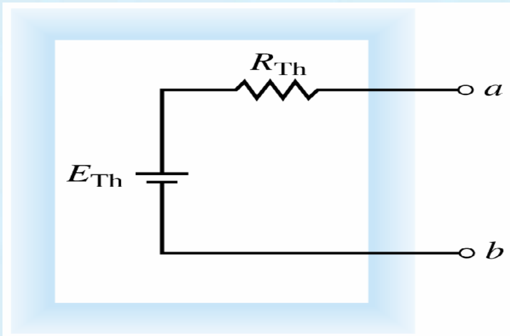
# Thévenin's Theorem




7

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# Thévenin's Theorem

- To convert to a Thévenin circuit
  - First identify and remove load from circuit
- Label resulting open terminals

8

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# Thévenin's Theorem

- Set all sources to zero
- Replace voltage sources with shorts, current sources with opens
- Determine Thévenin equivalent resistance as seen by open circuit






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# Thévenin's Theorem

- Replace sources and calculate voltage across open
- If there is more than one source
  – Superposition theorem could be used



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# Thévenin's Theorem

- Resulting open-circuit voltage is Thévenin equivalent voltage
- Draw Thévenin equivalent circuit, including load






# Norton's Theorem

- Similar to Thévenin circuit
- Any lumped linear bilateral network
  - May be reduced to a two-terminal circuit
  - Single current source and single shunt resistor

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# Norton's Theorem

- $R_N = R_{Th}$
- $I_N$ is Norton equivalent current

13

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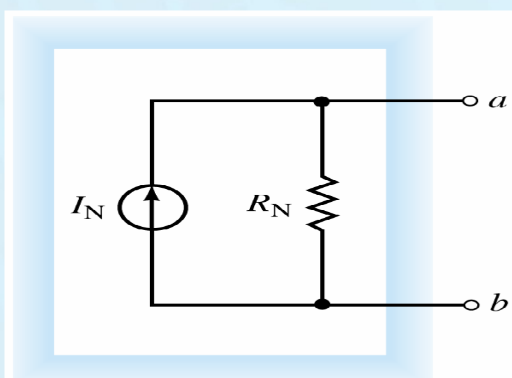
# Norton's Theorem




14

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## Norton's Theorem

- To convert to a Norton circuit
  - Identify and remove load from circuit
- Label resulting two open terminals
- Set all sources to zero






## Norton's Theorem

- Determine open circuit resistance
  - This is Norton equivalent resistance
- Note
  - This is accomplished in the same manner as Thévenin equivalent resistance

# Norton’s Theorem

- Replace sources and determine current that would flow through a short place between two terminals
- This current is the Norton equivalent current





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# Norton’s Theorem

- For multiple sources
  - Superposition theorem could be used
- Draw the Norton equivalent circuit
  - Including the load


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# Norton's Theorem

- Norton equivalent circuit
  - May be determined directly from a Thévenin circuit (or vice-versa) by using source transformation theorem




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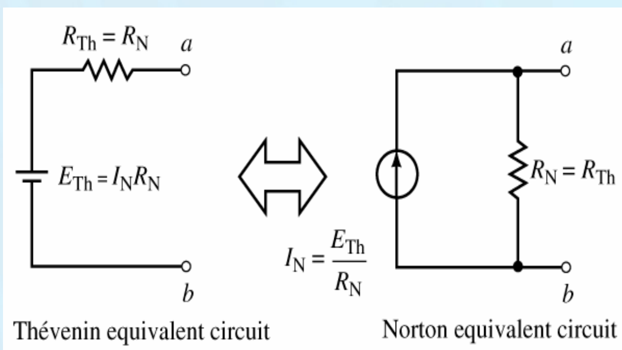
# Norton's Theorem



# Maximum Power Transfer

- Load should receive maximum amount of power from source
- Maximum power transfer theorem states
  - Load will receive maximum power from a circuit when resistance of the load is exactly the same as Thévenin (or Norton) equivalent resistance of the circuit

21

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# Maximum Power Transfer

- To calculate maximum power delivered by source to load
  - Use $P = V^2/R$
- Voltage across load is one half of Thévenin equivalent voltage

22

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# Maximum Power Transfer

- Current through load is one half of Norton equivalent current

$$P_{max} = \frac{E_{Th}^2}{4R_{Th}} = \frac{I_N^2 R_N}{4}$$





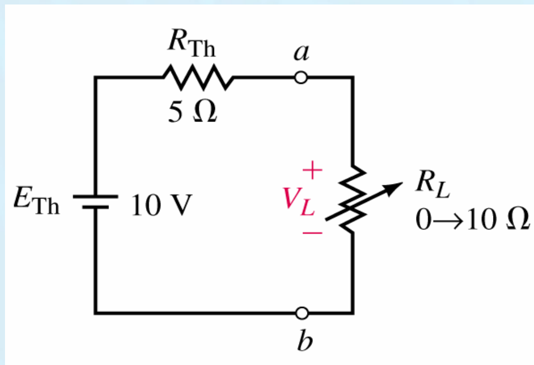




# Maximum Power Transfer

- Power across a load changes as load changes by using a variable resistance as the load

# Maximum Power Transfer







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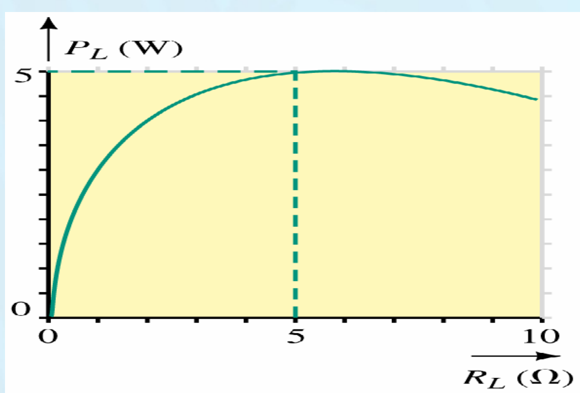
# Maximum Power Transfer



# Efficiency

- To calculate efficiency:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\%$$

$$\eta = \frac{\dfrac{E_{\text{Th}}{}^2}{4R_{\text{Th}}}}{\dfrac{E_{\text{Th}}{}^2}{2R_{\text{Th}}}} \times 100\% = 50\%$$

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# Substitution Theorem

- Any branch within a circuit may be replaced by an equivalent branch
  - Provided the replacement branch has same current voltage
- Theorem can replace any branch with an equivalent branch
- Simplify analysis of remaining circuit

# Substitution Theorem

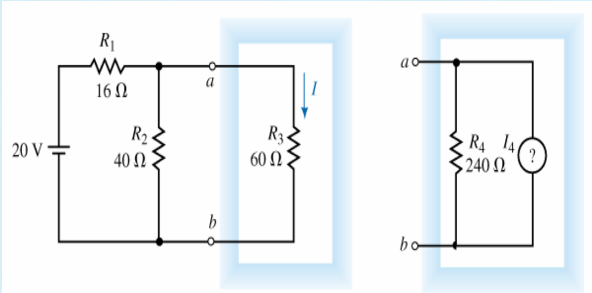
- Part of the circuit shown is to be replaced with a current source and a 240 Ω shunt resistor
  - Determine value of the current source








# Substitution Theorem




(a) (b)

# Millman’s Theorem

- Used to simplify circuits that have
  - Several parallel-connected branches containing a voltage source and series resistance
  - Current source and parallel resistance
  - Combination of both






# Millman’s Theorem

- Other theorems may work, but Millman’s theorem provides a much simpler and more direct equivalent

# Millman's Theorem

- Voltage sources
  - May be converted into an equivalent current source and parallel resistance using source transformation theorem
- Parallel resistances may now be converted into a single equivalent resistance





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# Millman's Theorem

- First, convert voltage sources into current sources
- Equivalent current, $I_{eq}$, is just the algebraic sum of all the parallel currents

# Millman’s Theorem

- Next, determine equivalent resistance, $R_{eq}$, the parallel resistance of all the resistors
- Voltage across entire circuit may now be calculated by:

$$E_{eq} = I_{eq}R_{eq}$$

35




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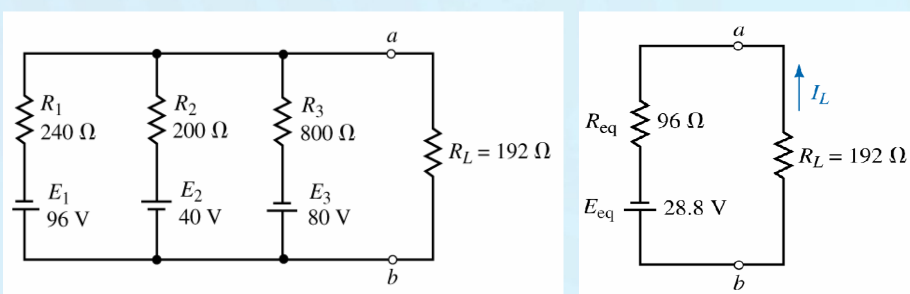
# Millman’s Theorem

- We can simplify a circuit as shown:




36

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# Reciprocity Theorem

- A voltage source causing a current *I* in any branch
  - May be removed from original location and placed into that branch






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# Reciprocity Theorem

- Voltage source in new location will produce a current in original source location
  - Equal to the original *I*



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## Reciprocity Theorem

- Voltage source is replaced by a short circuit in original location
- Direction of current must not change





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## Reciprocity Theorem

- A current source causing a voltage *V* at any node
  - May be removed from original location and connected to that node
- Current source in the new location
  - Will produce a voltage in original location equal to *V*


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# Reciprocity Theorem

- Current source is replaced by an open circuit in original location
- Voltage polarity cannot change