Chapter 21

Resonance

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Series Resonance

- Simple series resonant circuit
  - Has an ac source, an inductor, a capacitor, and possibly a resistor
- $\mathbf{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$
  - Resonance occurs when $X_L = X_C$
  - At resonance, $\mathbf{Z}_T = R$

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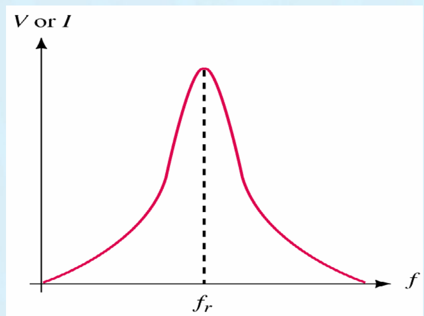
Series Resonance

- Response curves for a series resonant circuit



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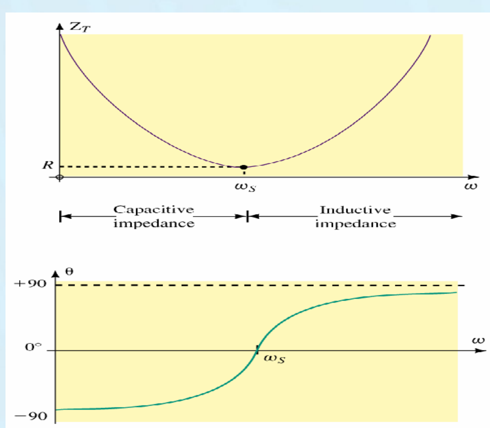
Series Resonance



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Series Resonance

- Since $X_L = \omega L = 2\pi fL$ and $X_C = 1/\omega C = 1/2\pi fC$ for resonance set $X_L = X_C$
  - Solve for the series resonant frequency f_s

$$\omega_s = \frac{1}{\sqrt{LC}} \text{(rad/sec)}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \text{(Hz)}$$

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Series Resonance

- At resonance
  - Impedance of a series resonant circuit is small and the current is large
- $\mathbf{I} = \mathbf{E}/\mathbf{Z}_T = E/R$

Series Resonance

- At resonance

$$V_R = IR$$
$$V_L = IX_L$$
$$V_C = IX_C$$

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Series Resonance

- At resonance, average power is $P = I^2R$
- Reactive powers dissipated by inductor and capacitor are I^2X
- Reactive powers are equal and opposite at resonance

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The Quality Factor, Q

- Q = reactive power/average power
  - Q may be expressed in terms of inductor or capacitor

$$Q_s = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

- For an inductor, $Q_{coil} = X_L / R_{coil}$

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The Quality Factor, Q

- Q is often greater than 1
  - Voltages across inductors and capacitors can be larger than source voltage

$$Q_s = \frac{IX}{IR} = \frac{V}{E}$$

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The Quality Factor, *Q*

- This is true even though the sum of the two voltages algebraically is zero

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Impedance of a Series Resonant Circuit

- Impedance of a series resonant circuit varies with frequency

$$\mathbf{Z}_T = R + j\omega L - \frac{j}{\omega C}$$

$$Z_T = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$

$$Z_T = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega RC}\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$$

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Bandwidth

- Bandwidth of a circuit
  - Difference between frequencies at which circuit delivers half of the maximum power
- Frequencies, f_1 and f_2
  - Half-power frequencies or the cutoff frequencies

Bandwidth

- A circuit with a narrow bandwidth
  - High selectivity
- If the bandwidth is wide
  - Low selectivity

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Bandwidth

- Cutoff frequencies
  - Found by evaluating frequencies at which the power dissipated by the circuit is half of the maximum power

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Bandwidth

$$I_{\text{hpf}} = \frac{I_{\max}}{\sqrt{2}}$$

$$\omega_1 = 2\pi f_1 = \frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\omega_2 = 2\pi f_2 = \frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

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Bandwidth

- From BW = $f_2 - f_1$
- BW = R/L
- When expression is multiplied by ω on top and bottom
  - BW = ω_S/Q (rad/sec) or BW = f_S/Q (Hz)

Series-to-Parallel Conversion

- For analysis of parallel resonant circuits
  - Necessary to convert a series inductor and its resistance to a parallel equivalent circuit

$$R_P = \frac{R_S^2 + X_{LS}^2}{R_S}$$

$$X_{LP} = \frac{R_S^2 + X_{LS}^2}{X_{LS}}$$

$$Q = \frac{X_{LS}}{R_S} = \frac{R_P}{X_{LP}}$$

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Series-to-Parallel Conversion

- If Q of a circuit is greater than or equal to 10
  - Approximations may be made
- Resistance of parallel network is approximately Q^2 larger than resistance of series network
  - $R_P \approx Q^2 R_S$
  - $X_{LP} \approx X_{LS}$

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Parallel Resonance

- Parallel resonant circuit
  - Has X_C and equivalents of inductive reactance and its series resistor, X_{LP} and R_S
- At resonance
  - $X_C = X_{LP}$

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Parallel Resonance

- Two reactances cancel each other at resonance
  - Cause an open circuit for that portion
- $\mathbf{Z}_T = R_P$ at resonance

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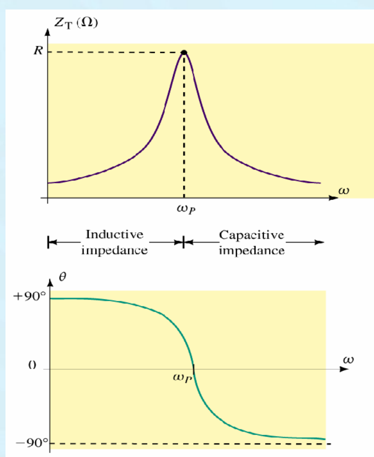
Parallel Resonance

- Response curves for a parallel resonant circuit



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Parallel Resonance

- From $X_C = X_{LP}$
  - Resonant frequency is found to be

$$f = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{L}}$$

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Parallel Resonance

- If $(L/C) >> R$
  - Term under the radical is approximately equal to 1
- If $(L/C) \geq 100R$
  - Resonant frequency becomes

$$f = \frac{1}{2\pi\sqrt{LC}}$$

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Parallel Resonance

- Because reactances cancel
  - Voltage is $V = IR$
- Impedance is maximum at resonance
  - $Q = R/X_C$
- If resistance of coil is the only resistance present
  - Circuit Q will be that of the inductor

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Parallel Resonance

- Circuit currents are

$$\mathbf{I}_R = \frac{\mathbf{V}}{\mathbf{R}}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{X_L} = Q_P I$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{X_C} = Q_P I$$

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Parallel Resonance

- Magnitudes of currents through the inductor and capacitor
  - May be much larger than the current source

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Bandwidth

- Cutoff frequencies are

$$\omega_1 = \frac{1}{2RC} - \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

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Bandwidth

- BW = $\omega_2 - \omega_1 = 1/RC$
- If $Q \geq 10$
  - Selectivity curve becomes symmetrical around ω_P

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Bandwidth

- Equation of bandwidth becomes

$$\mathrm{BW} = \frac{X_C \omega_P}{R}$$

$$\mathrm{BW} = \frac{\omega_P}{Q_P}$$

- Same for both series and parallel circuits

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