Chapter 25

Nonsinusoidal Waveforms

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Waveforms

- Used in electronics except for sinusoidal
- Any periodic waveform may be expressed as
  - Sum of a series of sinusoidal waveforms at different frequencies and amplitudes

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Waveforms

- Each sinusoidal components has a unique amplitude and frequency

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Waveforms

- These components have many different frequencies
  - Output may be greatly distorted after passing through a filter circuit

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Composite Waveforms

- Waveform made up of two or more separate waveforms
- Most signals appearing in electronic circuits
  - Comprised of complicated combinations of dc and sinusoidal waves

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Composite Waveforms

- Once a periodic waveform is reduced to the summation of sinusoidal waveforms
  - Overall response of the circuit can be found

Composite Waveforms

- Circuit containing both an ac source and a dc source
  - Voltage across the load is determined by superposition
- Result is a sine wave with a dc offset

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Composite Waveforms

- RMS voltage of composite waveform is determined as

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^{2} + V_{\text{ac}}^{2}}$$

- Referred to as true RMS voltage

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Composite Waveforms

- Waveform containing both dc and ac components
  - Power is determined by considering effects of both signals

Composite Waveforms

- Power delivered to load will be determined by

$$P_{out} = \frac{V_{rms}^2}{R_{load}}$$

Fourier Series

- Any periodic waveform
  - Expressed as an infinite series of sinusoidal waveforms
- Expression simplifies the analysis of many circuits that respond differently

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Fourier Series

- A periodic waveform can be written as:
  - $f(t) = a_0 + a_1\cos \omega t + a_2\cos 2\omega t + \cdots + a_n \cos n\omega t + \cdots + b_1\sin \omega t + b_2 \sin 2\omega t + \cdots + b_n \sin n\omega t + \cdots$

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Fourier Series

- Coefficients of terms of Fourier series
  - Found by integrating original function over one complete period

$$a_0 = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_{t_1}^{t_1+T} f(t)\sin(n\omega t)\,dt$$

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Fourier Series

- Individual components combined to give a single sinusoidal expression as:

$$a_n \cos nx + b_n \sin nx = a_n \sin(nx + 90°) + b_n \sin nx$$
$$= c_n \sin(nx + \theta)$$

where

$$c_n = \sqrt{a_n^2 + b_n^2}$$

and

$$\theta = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

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Fourier Series

- Fourier equivalent of any periodic waveform may be simplified to
  - $f(t) = a_0 + c_1\sin(\omega t + \theta_1) + c_2\sin(2\omega t + \theta_2) + \cdots$
- a_0 term is a constant that corresponds to average value
- c_n coefficients are amplitudes of sinusoidal terms

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Fourier Series

- Sinusoidal term with $n = 1$
  - Same frequency as original waveform
- First term
  - Called fundamental frequency

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Fourier Series

- All other frequencies are integer multiples of fundamental frequency
- These frequencies are harmonic frequencies or simply harmonics

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Fourier Series

- Pulse wave which goes from 0 to 1, then back to 0 for half a cycle, will have a series given by

$$v(t) = 0.5 + \frac{2}{\pi}\sum_{n}^{\infty}\frac{\sin(n\omega t)}{n}$$

$$n = 1,\, 3,\, 5, \cdots, \infty$$

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Fourier Series

- Average value
  - $a_0 = 0.5$
- It has only odd harmonics
- Amplitudes become smaller

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Even Symmetry

- Symmetrical waveforms
  - Around vertical axis have even symmetry
- Cosine waveforms
  - Symmetrical about this axis
  - Also called cosine symmetry

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Even Symmetry

- Waveforms having even symmetry will be of the form $f(-t) = f(t)$
- A series with even symmetry will have only cosine terms and possibly a constant term

Odd Symmetry

- Odd symmetry
  - Waveforms that overlap terms on opposite sides of vertical axis if rotated 180°
- Sine symmetry
  - Sine waves that have this symmetry

Odd Symmetry

- Waveforms having odd symmetry will always have the form $f(-t) = -f(t)$
- Series will contain only sine terms and possibly a constant term

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Half-Wave Symmetry

- Portion of waveform below horizontal axis is mirror image of portion above axis

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Half-Wave Symmetry

- These waveforms will always be of the form

$$f\left(t+\frac{T}{2}\right)=-f(t)$$

- Series will have only odd harmonics and possibly a constant term

Shifted Waveforms

- If a waveform is shifted along the time axis
  - Necessary to include a phase shift with each of the sinusoidal terms
- To determine the phase shift
  - Determine period of given waveforms

Shifted Waveforms

- Select which of the known waveforms best describes the given wave

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Shifted Waveforms

- Determine if given waveform leads or lags a known waveform
- Calculate amount of phase shift from $\phi = (t/T) \cdot 360°$
- Write resulting Fourier expression for given waveform

Shifted Waveforms

- If given waveform leads the known waveform
  - Add phase angle
  - If it lags, subtract phase angle

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Frequency Spectrum

- Waveforms may be shown as a function of frequency
  - Amplitude of each harmonic is indicated at that frequency

Frequency Spectrum

- True RMS voltage of composite waveform is determined by considering RMS value at each frequency

$$V_{\text{rms}} = \sqrt{V_{\text{dc}}^{2} + V_1^{2} + V_2^{2} + V_3^{2} + \cdots}$$

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Frequency Spectrum

- If a waveform were applied to a resistive element
  - Power would be dissipated as if each frequency had been applied independently
- Total power is determined as sum of individual powers

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Frequency Spectrum

- To calculate power
  - Convert all voltages to RMS
- Frequency spectrum may then be represented in terms of power

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Frequency Spectrum

- Power levels and frequencies of various harmonics of a periodic waveform may be measured with a spectrum analyzer
- Some spectrum analyzers display either voltage levels or power levels

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Frequency Spectrum

- When displaying power levels
  - 50-Ω reference load is used
- Horizontal axis is in hertz
  - Vertical axis is in dB

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Circuit Response to a Nonsinusoidal Waveform

- When a waveform is applied to input of a filter
  - Waveform may be greatly modified
- Various frequencies may be blocked by filter

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Circuit Response to a Nonsinusoidal Waveform

- A composite waveform passed through a bandpass filter
  - May appear as a sine wave at desired frequency
- Method is used to provide frequency multiplication