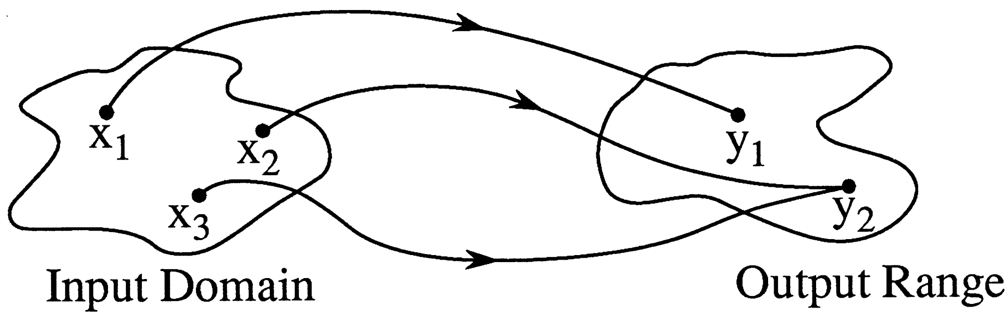
1.2 SYSTEMS

1. System properties

2. Convolution

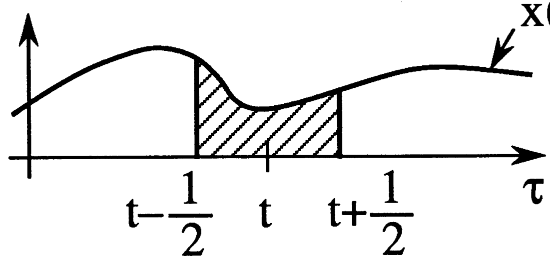
3. Frequency response

A *system* is a mapping which assigns to each signal x(t) in the *input domain* a unique signal y(t) in the *output range*.



Examples

1. (CT) $$y(t) = \int_{t-1/2}^{t+1/2} x(\tau)d\tau$$



let $x(t) = \sin(2\pi ft)$

$$y(t) = \int_{t-1/2}^{t+1/2} \sin(2\pi f\tau)d\tau$$

$$y(t) = -\frac{1}{2\pi f}\cos(2\pi ft)\Bigg|_{t-1/2}^{t+1/2}$$

$$= -\frac{1}{2\pi f}\{\cos[2\pi f(t + 1/2)] - \cos[2\pi f(t - 1/2]\}$$

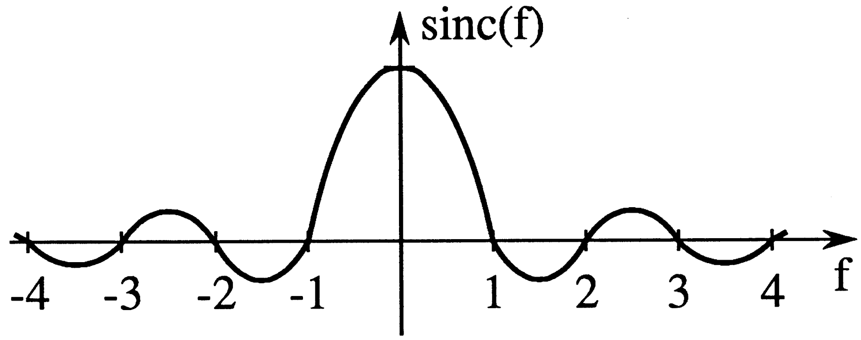
use $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$$y(t) = -\frac{1}{2\pi f}\{\cos(2\pi ft)\cos(\pi f) - \sin(2\pi ft)\sin(\pi f)$$

$$- [\cos(2\pi ft)\cos(\pi f) + \sin(2\pi ft)\sin(\pi f)]\}$$

$$y(t) = \frac{\sin(\pi f)}{\pi f} \sin(2\pi f t)$$

$$= \text{sinc}(f)\sin(2\pi f t)$$

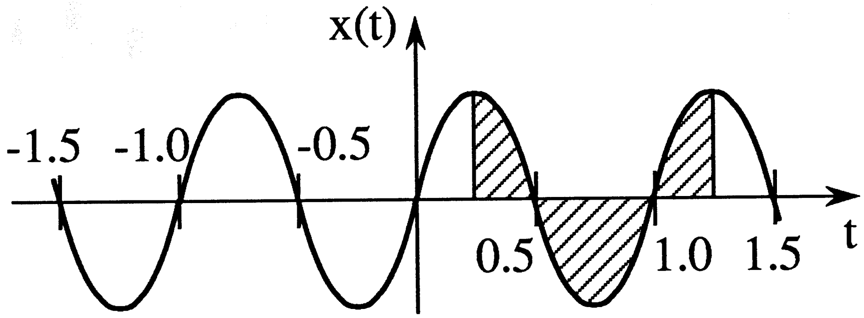


Look at particular values of f

$$f = 0: \quad x(t) \equiv 0$$

$$y(t) \equiv 0$$

$$f = 1: \quad x(t) = \sin(2\pi t)$$

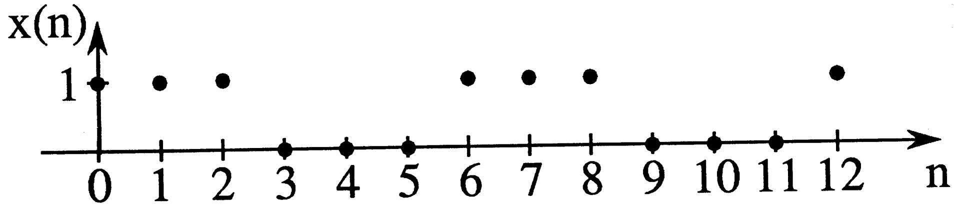


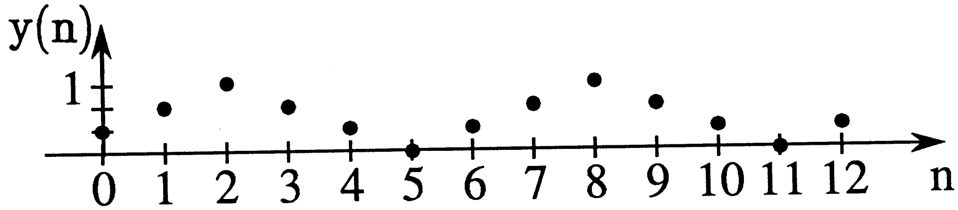
$$\mathrm{sinc}(1) = 0$$

$$y(t) \equiv 0$$

2. (DT) $y(n) = \frac{1}{3}\,[x(n) + x(n-1) + x(n-2)]$

n	...	0	1	2	3	4	5	6	...
x(n)	...	1	1	1	0	0	0	1	...
y(n)	...	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	...





Effect of filter on input:

1. widened, smoothed, or smeared each pulse

2. delayed pulse train by one sample time

Need a general theory to describe what is happening here.