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THE ESTIMATION OF SPECTRAL MOMENTS

BY

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THE ESTIMATION OF SPECTRAL MOMENTS

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The results of an investigation that considered the real time estimation of certain shape parameters of the power spectral density of a narrow band random process are presented. Systems are developed which directly estimate f_a, the power mean frequency, and B, the standard deviation of the spectrum. The equivalence of these estimators to the classical estimators is demonstrated and simple implementations, due to the omission of the intermediate computation of spectrum estimates, are presented.

Statistical analyses are made for both estimators under the assumption of long time averaging and a gaussian input process. It is demonstrated that both estimators are asymptotically unbiased and that their mean square errors tend to zero, with order T^{-1}, as the averaging time goes to infinity.

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LIST OF SYMBOLS AND ABBREVIATIONS

Symbol	Definition	Page
A	instantaneous envelope of the random process	4
α	real part of the complex envelope of the random process after passing through an infinite time window $\rho(t)$	26
B	RMS spectral bandwidth of the random process	1
$\hat{B}$	estimate of B	3
B^2	mean square spectral bandwidth of the random process	1
B_p^2	mean square spectral bandwidth of the time-limited window function $p(t)$	23
B_ρ^2	mean square spectral bandwidth of the smooth, infinite window function $\rho(t)$	24
B_T^2	spreading of the random process' bandwidth on a particular measurement which is due to the discontinuities of the window function	31
$\overline{B_T^2}$	average spreading of the bandwidth due to the discontinuities of the window function	31
β	imaginary part of the complex envelope of the random process after passing through an infinite time window $\rho(t)$	26
D	doppler spread parameter equal to $B/2$	3
$\hat{D}_1$	estimate of D using the quadrature components of the complex envelope of the random process	3
$\hat{D}_2$	estimate of D using only knowledge of the instantaneous envelope of the random process	3

LIST OF SYMBOLS AND ABBREVIATIONS (Continued)

Symbol	Definition	Page
$\delta(t)$	the Dirac delta function	26
$\Delta s(f)$	error in a particular estimate of the power spectrum	13
Disc.	discriminator	30
$E(\ )$	the ensemble average of the quantity within the parenthesis	10
$\epsilon(\ )$	the mean square error of the quantity within the parenthesis	10
f_a	the power mean frequency	1
$\hat{f}_a$	estimate of f_a	3
$\widehat{f_a P}$	estimate of the product of the power mean frequency and the total power	5
f_c	correlation frequency	15
f_i	instantaneous frequency (Hz)	28
f_0	transmitter carrier frequency	30
$G_W(f)$	the unit gate function of the width W	4
γ	envelope detector efficiency	4
$H_1(\omega), H_2(\omega)$	filter characteristics in specialized discriminator	4
Hz	Hertz, cycles per second	53
$\int_f (\ )\, df$	the integral over all values of f of the quantity within the parenthesis	1
k_1, k_2	constants of proportionality	16
M_k	kth moment of the spectral estimate	38
$\overline{M_k}$	ensemble average of M_k	38
m_k	error in a particular value of M_k	38
ω_i	instantaneous frequency (radians/second)	6

LIST OF SYMBOLS AND ABBREVIATIONS (Continued)

Symbol	Definition	Page
ω_o	transmitter carrier frequency (radians/second)	29
P	total power in the random process	6
$\hat{P}$	estimate of P	24
p(t)	time-limited window function	22
$\xi(t)$	complex envelope of the random process after passing through an infinite time window	24
PDF	probability density function	6
PSD	power spectral density	1
$R_\xi(t)$	time autocorrelation function of the complex envelope of the random process after passing through the infinite time window	60
RMS	root mean square	1
$\rho(t)$	infinite time window function	23
S(f)	power spectral density	1
$S_\xi(f)$	power spectral density of the process $\xi(t)$	40
$\sigma_s^2(f)$	mean square error in the spectral estimate	15
T	the observation time	3
$\theta(t)$	instantaneous phase of the random process	28
x	real part of the complex envelope of the random process	3
$\dot{x}$	derivative of x with respect to time	3
y	imaginary part of the complex envelope of the random process	3

LIST OF SYMBOLS AND ABBREVIATIONS (Continued)

Symbol	Definition	Page
$z(t)$	the complex envelope of the random process	1
$Z_T(2\pi f)$	the Fourier transform of $z(t)p(t)$	22